Homework Answer

Angle properties of circle

1. **a** x = 30° (theorem 2)
   b x = 25°, y = 25° (theorem 2 for both angles)
   c x = 32° (theorem 2)
   d x = 40°, y = 40° (theorem 2 for both angles)
   e x = 60° (theorem 1)
   f x = 40° (theorem 1)
   g x = 84° (theorem 1)
   h x = 50° (theorem 2); y = 100° (theorem 1)

2. **a** s = 90°, r = 90° (theorem 3 for both angles)
   b u = 90° (theorem 4); t = 90° (theorem 3)
   c m = 90°, n = 90° (theorem 3 for both angles)
   d x = 52° (theorem 3 and angle sum in a triangle = 180°)
   e x = 90° (theorem 4)
   f x = 90° (theorem 4); y = 15° (angle sum in a triangle = 180°)

3.

a $\alpha = 120^\circ$ (sum of angles about a point is 360°),
$\beta = 60^\circ$ (angle at the circumference is half the angle at the center standing on the same arc)

b $\theta = 30^\circ$ ($\angle AOB = 60^\circ$, sum of angles about a point is 360° and angle at the circumference is half the angle at the center standing on the same arc, so θ is half of $\angle AOB$)

c $\theta = 220^\circ$ ($\angle SOR = 140^\circ$, angle at the center is twice the angle at the circumference standing on the same arc, and $\theta + \angle SOR = 360^\circ$, sum of angles about a point is 360°)

d $\alpha = \beta = 40^\circ$ (any angle at the circumference is half the angle at the center standing on the same arc)

e $\theta = 320^\circ$ ($\angle SOR = 40^\circ$, angle at the center is twice the angle at the circumference standing on the same arc, and $\theta + \angle SOR = 360^\circ$, sum of angles about a point is 360°)

$\alpha = 20^\circ$ (angle at the circumference is half the angle at the center standing on the same arc, $\angle SOR = 40^\circ$)

f $\alpha = 100^\circ$ ($OR = OQ$ radii, so base angles of an isosceles triangle are both 40°, sum of angles in a triangle is 180°),
$\beta = 140^\circ$ (sum of angles about a point is 360°),
$\gamma = 20^\circ$ ($OR = OP$ radii, γ is a base angle of an isosceles triangle)

g $\alpha = 80^\circ$ (angle at the circumference is half the angle at the center standing on the same arc),
$\beta = 200^\circ$ (sum of angles about a point is 360°),
$\gamma = 100^\circ$ (angles at the circumference are half the angle at the center standing on the same arc)

h $\alpha = 60^\circ$ ($\angle DAB = 90^\circ$, Thales' theorem, so $\alpha + 30^\circ = 90^\circ$),
$\beta = 60^\circ$ ($OA = OB$ radii, so $\alpha = \beta$, base angles of an isosceles triangle),
$\gamma = 30^\circ$ ($\angle ABC = 90^\circ$, Thales' theorem, so $\gamma + \beta = 90^\circ$)

i $\alpha = \beta = 45^\circ$ (both are base angles in isosceles triangles with the third angle 90°)

4.

a $\alpha = 90^{\circ}$ (Thales' theorem),

$\beta = 10^{\circ}$ (alternate angles, $AB \parallel FG$)

b $\alpha = 60^{\circ}$ ($OP = OA = AP$, so the triangle is equilateral),

$\beta = 30^{\circ}$ (angle at the circumference is half the angle at the center standing on the same arc)

c $\alpha = 20^{\circ}$ (alternate angles, $PO \parallel QR$),

$\gamma = 40^{\circ}$ (angle at the center is twice the angle at the circumference standing on the same arc),

$\beta = 40^{\circ}$ (alternate angles, $PO \parallel OR$)

d $\alpha = 220^{\circ}$ (sum of angles about a point),

$\beta = 110^{\circ}$ (angle at the circumference is half the angle at the center standing on the same arc),

$\gamma = 60^{\circ}$ (sum of angles in a quadrilateral is 360°)

e $\alpha = 200^{\circ}$ (sum of angles about a point),

$\beta = 100^{\circ}$ (angle at the circumference is half the angle at the center standing on the same arc),

$\gamma = 80^{\circ}$ (co-interior angles, $PQ \parallel OR$)

f $\alpha = 100^{\circ}$ (angle at the circumference is half the angle at the center standing on the same arc),

$\beta = 60^{\circ}$ (construct OA, as $OB = AB = OA$, the triangle is equilateral),

$\gamma = 40^{\circ}$ (sum of angles in a quadrilateral is 360°)

Tangents to a circle

1. **a** $x = z = 90°$ (theorem 4); $y = w = 20°$ (theorem 5 and angle sum in a triangle = 180°)
   b $s = r = 90°$ (theorem 4); $t = 140°$ (angle sum in a quadrilateral = 360°)
   c $x = 20°$ (theorem 5); $y = z = 70°$ (theorem 4 and angle sum in a triangle = 180°)
   d $s = y = 90°$ (theorem 4); $x = 70°$ (theorem 5);
   $r = z = 20°$ (angle sum in a triangle = 180°)
   e $x = 70°$ (theorem 4 and angle sum in a triangle = 180°); $y = z = 20°$ (angle sum in a triangle = 180°)
   f $x = y = 75°$ (theorem 4 and angle sum in a triangle = 180°); $z = 75°$ (theorem 1)

2.

a $x = 5$ (tangents to a circle from an external point have equal length),
$\alpha = 70^\circ$ (base angle of an isosceles triangle),
$\beta = 40^\circ$ (sum of angles in a triangle),
$\gamma = 20^\circ$ ($\angle OSP = 90^\circ$)

b $x = 8$ (tangents to a circle from an external point have equal length),
$\alpha = 70^\circ$ (base angle of an isosceles triangle),
$\theta = 140^\circ$ ($\angle T = \angle S = 90^\circ$, sum of angles in a quadrilateral)

c $x = 2$ (tangents to a circle from an external point have equal length),
$y = 3, z = 3$ (tangents to a circle from an external point have equal length)

d $x = 7$ ($SQ = 4$, tangents to a circle from an external point have equal length, so $RS = 7$)

e $x = 9$ ($SB = 4$ (equal tangents), $SP = 14$ and $TP = 14$ (equal tangents), $TA = 5$ (equal tangents), $x = 14 - 5$)

f $\alpha = 100^\circ$ (reflex $\angle SOT = 260^\circ$, angle at the center is twice the angle at the circumference standing on the same arc, sum of angles at point O),
$\beta = 70^\circ$ (sum of angles in a quadrilateral),
$\gamma = 20^\circ$ ($\angle PSO = 90^\circ$)

Intersecting chords, secants and tangents

1. **a** $m = 3$

   b $m = 3$

   c $m = 6$

   d $n = 1$

   e $m = 7.6$

   f $n = 13$

   g $x = 12$

   h $x = 5\frac{1}{2}$

   i $x = 2$

   j $x = 30$

   k $x = 2$

   l $x = 2\sqrt{2}$

   m $x = 1.2$

   n $x = 5$

   o $x = \sqrt{30}$

   p $x = 6$

   q $x = 7.5$

   r $x = 5$