Homework Answer

Chapter 3 Logarithms

3A Introducing logarithms

1.

$\log_2(64) = 6 \Leftrightarrow 2^6 = 64$	$\log_5(25) = 2 \Leftrightarrow 5^2 = 25$	$\log_{10}(0.1) = -1 \Leftrightarrow 10^{-1} = 0.1$	$\log_4(16) = 2 \Leftrightarrow 4^2 = 16$
$\log_9(3) = \frac{1}{2} \Leftrightarrow 9^{1/2} = 3$	$\log_8(64) = 2 \Leftrightarrow 8^2 = 64$	$\log_{16}(2) = \frac{1}{4} \Leftrightarrow 16^{1/4} = 2$	$\log_3(81) = 4 \Leftrightarrow 3^4 = 81$
$\log_{25}(5) = \frac{1}{2} \Leftrightarrow 25^{1/2} = 5$	$\log_{16}(4) = \frac{1}{2} \Leftrightarrow 16^{1/2} = 4$	$\log_4(8) = \frac{3}{2} \Leftrightarrow 4^{3/2} = 8$	$\log_4(\frac{1}{16}) = -2 \Leftrightarrow 4^{-2} = \frac{1}{16}$
$4^3 = 64 \Leftrightarrow \log_4(64) = 3$	$2^{-3} = \frac{1}{8} \Leftrightarrow \log_2(\frac{1}{8}) = -3$	$3^{-2} = \frac{1}{9} \Leftrightarrow \log_3(\frac{1}{9}) = -2$	$10^1 = 10 \Leftrightarrow \log_{10}(10) = 1$
$9^{1/2} = 3 \Leftrightarrow \log_9(3) = \frac{1}{2}$	$5^{-3} = \frac{1}{125} \Leftrightarrow \log_5(\frac{1}{125}) = -3$	$8^{4/3} = 16 \Leftrightarrow \log_8(16) = \frac{4}{3}$	$(\frac{1}{4})^{-2} = 16 \Leftrightarrow \log_{\frac{1}{4}}(16) = -2$
$7^2 = 49 \Leftrightarrow \log_7(49) = 2$	$25^{3/2} = 125 \Leftrightarrow \log_{25}(125) = \frac{3}{2}$	$6^2 = 36 \Leftrightarrow \log_6(36) = 2$	$2^4 = 16 \Leftrightarrow \log_2(16) = 4$

2.

$\log_2(32) = 5$	$\log_8(1) = 0$	$\log_{10}(100) = 2$	$\log_4(\frac{1}{16}) = -2$
$\log_3(81) = 4$	$\log_9(3) = \frac{1}{2}$	$\log_{25}(125) = \frac{3}{2}$	$\log_3(243) = 5$
$\log_2(\frac{1}{8}) = -3$	$\log_4(2) = \frac{1}{2}$	$\log_2(1024) = 10$	$\log_{27}(3) = \frac{1}{3}$
$\log_{36}(6) = \frac{1}{2}$	$\log_{49}(7) = \frac{1}{2}$	$\log_{12}(144) = 2$	$\log_{100}(0.1) = -\frac{1}{2}$
$\log_5(\frac{1}{5}) = -1$	$\log_8(4) = \frac{2}{3}$	$\log_{64}(8) = \frac{1}{2}$	$\log_{16}(\frac{1}{2}) = -\frac{1}{4}$

3.

$\log_2(16) = x$ $\boxed{x = 4}$	$\log_5(125) = x$ $\boxed{x = 3}$	$\log_4(64) = x$ $\boxed{x = 3}$	$\log_9(81) = 2$ (already 2)
$\log_2(\frac{1}{8}) = x$ $\boxed{x = -3}$	$\log_5(x) = 2$ $\boxed{x = 25}$	$\log_3(x) = 5$ $\boxed{x = 243}$	$\log_2(x) = -3$ $\boxed{x = \frac{1}{8}}$
$\log_6(216) = x$ $\boxed{x = 3}$	$\log_{10}(x) = -2$ $\boxed{x = \frac{1}{100}}$	$\log_3(\frac{1}{9}) = x$ $\boxed{x = -2}$	$\log_4(x) = \frac{1}{2}$ $\boxed{x = 2}$
$\log_5(\frac{1}{25}) = x$ $\boxed{x = -2}$	$\log_8(x) = 2$ $\boxed{x = 64}$	$\log_2(256) = x$ $\boxed{x = 8}$	$\log_6(x) = 1$ $\boxed{x = 6}$
$\log_8(\frac{1}{8}) = x$ $\boxed{x = -1}$	$\log_{125}(x) = 1$ $\boxed{x = 125}$	$\log_9(3) = x$ $\boxed{x = \frac{1}{2}}$	$\log_5(1) = x$ $\boxed{x = 0}$

3B. Laws of logarithms

1.

$\log_2(4) + \log_2(8) = 2 + 3 = 5$	$\log_3(3) + \log_3(9) = 1 + 2 = 3$	$\log_4(2) + \log_4(8) = \frac{1}{2} + \frac{3}{2} = 2$
$\log_5(25) + \log_5(\frac{1}{5}) = 2 + (-1) = 1$	$\log_6(36) + \log_6(6) = 2 + 1 = 3$	$\log_4(4) + \log_4(16) = 1 + 2 = 3$
$\log_2(\frac{1}{2}) + \log_2(8) = -1 + 3 = 2$	$\log_3(1) + \log_3(3) = 0 + 1 = 1$	$\log_8(8) + \log_8(64) = 1 + 2 = 3$
$\log_2(32) + \log_2(64) = 5 + 6 = 11$	$\log_3(9) + \log_3(\frac{1}{3}) = 2 + (-1) = 1$	$\log_{10}(10) + \log_{10}(100) = 1 + 2 = 3$
$\log_2(8) - \log_2(4) = 3 - 2 = 1$	$\log_3(27) - \log_3(3) = 3 - 1 = 2$	$\log_4(64) - \log_4(16) = 3 - 2 = 1$
$\log_5(125) - \log_5(25) = 3 - 2 = 1$	$\log_{10}(1000) - \log_{10}(100) = 3 - 2 = 1$	$\log_2(32) - \log_2(8) = 5 - 3 = 2$
$\log_4(1) - \log_4(\frac{1}{4}) = 0 - (-1) = 1$	$\log_5(25) - \log_5(\frac{1}{5}) = 2 - (-1) = 3$	$\log_2(\frac{1}{2}) - \log_2(\frac{1}{4}) = -1 - (-2) = 1$
$\log_{10}(100) - \log_{10}(0.01) = 2 - (-2) = 4$	$\log_2(64) - \log_2(\frac{1}{8}) = 6 - (-3) = 9$	$\log_3(9) - \log_3(\frac{1}{3}) = 2 - (-1) = 3$

2.

$2\log_2(3) + \log_2(4) = \log_2(36)$	$\log_3(8) + 2\log_3(9) = \log_3(648)$	$3\log_2(2) + \log_2(8) = 6$
$2\log_5(5) + \log_5(25) = 4$	$2\log_4(2) + \log_4(8) = \frac{5}{2}$	$3\log_3(3) + \log_3(9) = 5$
$3\log_5(2) + \log_5(4) = \log_5(32)$	$2\log_2(5) + \log_2(5) = \log_2(125)$	$4\log_2(10) - \log_2(100) = \log_2(100)$
$2\log_3(4) - \log_3(2) = \log_3(8)$	$3\log_5(25) - 2\log_5(5) = 4$	$4\log_2(2) - \log_2(16) = 0$
$5\log_3(3) - 2\log_3(9) = 1$	$2\log_4(16) - \log_4(4) = 3$	$3\log_6(6) - \log_6(36) = 1$
$2\log_7(7) + \log_7(49) = 4$	$\log_5(125) + 2\log_5(\frac{1}{5}) = 1$	$2\log_4(8) + \log_4(2) = \frac{7}{2}$

3C. Solving exponential equations using logarithms

1.

$2^x = 7$ $x \approx 2.807$	$2^x = 20$ $x \approx 4.322$	$3^x = 50$ $x \approx 3.560$	$4^x = 100$ $x \approx 3.322$
$5^x = 12$ $x \approx 1.544$	$9^x = 27$ $x = 1.500$	$8^x = 1000$ $x \approx 3.322$	$10^x = 30$ $x \approx 1.477$
$3^x = 2$ $x \approx 0.631$	$5^x = 10$ $x \approx 1.431$	$7^x = 12$ $x \approx 1.232$	$9^x = 18$ $x \approx 1.315$
$(1.5)^x = 3$ $x \approx 2.709$	$(2.3)^x = 10$ $x \approx 2.764$	$(0.4)^x = 0.2$ $x \approx 1.757$	$(0.5)^x = 0.25$ $x = 2.000$
$(0.6)^x = 0.2$ $x \approx 3.151$	$(0.8)^x = 0.64$ $x = 2.000$	$(1.2)^x = 4$ $x \approx 7.606$	$(2.7)^x = 6$ $x \approx 1.804$
$(0.7)^x = 0.49$ $x = 2.000$	$(1.01)^x = 1.3$ $x \approx 26.385$	$(2.5)^x = 5$ $x \approx 1.757$	$(6.3)^x = 8.8$ $x \approx 1.182$
$10 \times 2^x = 80$ $2^x = 8 \Rightarrow x =$ 3.000	$3 \times 5^x = 90$ $5^x = 30 \Rightarrow x \approx$ 2.114	$0.5 \times 4^x = 8$ $4^x = 16 \Rightarrow x =$ 2.000	$4 \times (0.8)^x = 0.8$ $(0.8)^x = 0.2 \Rightarrow x \approx$ 7.215
$2.5 \times (1.5)^x = 12$ $(1.5)^x = 4.8 \Rightarrow$ $x \approx 3.871$	$7 \times (1.4)^x = 70$ $(1.4)^x = 10 \Rightarrow$ $x \approx 6.842$	$2 \times (0.7)^x = 1$ $(0.7)^x = 0.5 \Rightarrow$ $x \approx 1.944$	$3 \times (0.3)^x = 0.1$ $(0.3)^x = 0.0333 \Rightarrow$ $x \approx 2.826$

3D Introducing polynomials

1.

$4x - 1$ Degree $= 1$	$x^3 + x - 7$ Degree $= 3$	$2x^5 + 9$ Degree $= 5$
$3 - 8x^2 + 10x^7$ Degree $= 7$	$4y^2 + 3y + 1$ Degree $= 2$	$7z^4 - 2z + 9$ Degree $= 4$
$-2t^6 + 9t^2$ Degree $= 6$	$3u^4 + 8u^3 - 10$ Degree $= 4$	$11v - 17$ Degree $= 1$

2.

a. 8

b. x

c. 0

d. 2

e. 18x^4

f. -6x^8

3.

a. 8

b. X

c. 3

d. 5

e. -6x^3

f. 9x^8

4.

(a) $P(2)$

$$P(2) = -10(2) + 5(2^3) + (2^2) = -20 + 5(8) + 4 = -20 + 40 + 4 = 24.$$

(b) $P(-2)$

$$P(-2) = -10(-2) + 5((-2)^3) + (-2)^2 = 20 + 5(-8) + 4 = 20 - 40 + 4 = -16.$$

(c) $P(a)$

$$P(a) = -10a + 5a^3 + a^2.$$

(d) $P(-a)$

$$P(-a) = -10(-a) + 5((-a)^3) + (-a)^2 = 10a + 5(-a^3) + a^2 = 10a - 5a^3 + a^2.$$

(e) $P(-2a)$

$$P(-2a) = -10(-2a) + 5((-2a)^3) + (-2a)^2 = 20a + 5(-8a^3) + 4a^2 = 20a - 40a^3 + 4a^2.$$

(f) $P(y)$

$$P(y) = -10y + 5y^3 + y^2.$$

(g) $P(ab)$

$$P(ab) = -10(ab) + 5((ab)^3) + (ab)^2 = -10ab + 5a^3b^3 + a^2b^2.$$

(h) $P(-ab)$

$$P(-ab) = -10(-ab) + 5((-ab)^3) + (-ab)^2 = 10ab + 5(-a^3b^3) + a^2b^2 = 10ab - 5a^3b^3 + a^2b^2.$$

5.

(a) $P(1)$

$$P(1) = 1 - 2(1) + 3(1^2) + 2(1^3) = 1 - 2 + 3 + 2 = 4.$$

(b) $P(-1)$

$$P(-1) = 1 - 2(-1) + 3((-1)^2) + 2((-1)^3) = 1 + 2 + 3 - 2 = 4.$$

(c) $P(2)$

$$P(2) = 1 - 2(2) + 3(2^2) + 2(2^3) = 1 - 4 + 3(4) + 2(8) = 1 - 4 + 12 + 16 = 25.$$

(d) $P(-2)$

$$P(-2) = 1 - 2(-2) + 3((-2)^2) + 2((-2)^3) = 1 + 4 + 3(4) + 2(-8) = 1 + 4 + 12 - 16 = 1.$$

(e) $P(a)$

$$P(a) = 1 - 2a + 3a^2 + 2a^3.$$

(f) $P(-a)$

$$P(-a) = 1 - 2(-a) + 3((-a)^2) + 2((-a)^3) = 1 + 2a + 3a^2 - 2a^3.$$

(g) $P(ab)$

$$P(ab) = 1 - 2(ab) + 3((ab)^2) + 2((ab)^3) = 1 - 2ab + 3a^2b^2 + 2a^3b^3.$$

(h) $P(-ab)$

$$P(-ab) = 1 - 2(-ab) + 3((-ab)^2) + 2((-ab)^3) = 1 + 2ab + 3a^2b^2 - 2a^3b^3.$$

3E Adding, subtracting and multiplying polynomials

1.

1(a) $(2x^2+3x-4)+(x^2+5x^3-1)=5x^3+3x^2+3x-5$	1(b) $(x+x^2+1)+(x^4-x+2)=x^4+x^2+3$
1(c) $(5x^3-x+1)+(x^3-4x^2+2)=6x^3-4x^2-x+3$	1(d) $(7x^4-3x^2+5)+(x^5+2y^4-1)=x^5+7x^4+2y^4-3x^2+4$
1(e) $(4x^3+3x^2-2)-(x^4-x^2+1)=-x^4+4x^3+4x^2-3$	1(f) $(x^2+x-1)-(3x^2-5x+4)=-2x^2+6x-5$
1(g) $(6x^3+2x-1)-(x^3+3x^2-2)=5x^3-3x^2+2x+1$	1(h) $(x^4+4x^2-5)-(2x^4-x^2+3)=-x^4+5x^2-8$

2.

2(a) $(x-2)(x+3)(x+1)=x^3+2x^2-5x-6$	2(b) $(x+1)(x+2)(x+3)=x^3+6x^2+11x+6$
2(c) $(x-1)(x-2)(x+1)=x^3-2x^2-x+2$	2(d) $(x-1)(x+7)\,x=x^3+6x^2-7x$
2(e) $(2x+1)(x-3)(x-2)=2x^3-9x^2+7x+6$	2(f) $(3x-2)(x+2)(x+5)=3x^3+19x^2+16x-20$

3.

3(a) $(x+1)^3=x^3+3x^2+3x+1$	3(b) $(x-2)^3=x^3-6x^2+12x-8$
3(c) $(x+5)^3=x^3+15x^2+75x+125$	3(d) $(x-6)^3=x^3-18x^2+108x-216$

4.

(a) $P(x) + Q(x)$

$$\begin{aligned} P(x) + Q(x) &= (x^2 - 2x + 1) + (x^2 + 2x) \\ &= x^2 + x^2 + (-2x + 2x) + 1 \\ &= 2x^2 + 1. \end{aligned}$$

(b) $Q(x) - P(x)$

$$\begin{aligned} Q(x) - P(x) &= (x^2 + 2x) - (x^2 - 2x + 1) \\ &= x^2 - x^2 + 2x + 2x - 1 \\ &= 4x - 1. \end{aligned}$$

(c) $P(x)\,Q(x)$

Multiply each term of $P(x)$ by each term of $Q(x)$:

$$\begin{aligned} P(x)\,Q(x) &= (x^2 - 2x + 1) \cdot (x^2 + 2x) \\ &= x^2 \cdot x^2 + x^2 \cdot 2x \;-\; 2x \cdot x^2 \;-\; 2x \cdot 2x \;+\; 1 \cdot x^2 \;+\; 1 \cdot 2x \\ &= x^4 + 2x^3 - 2x^3 - 4x^2 + x^2 + 2x \\ &= x^4 + \left(2x^3 - 2x^3\right) + \left(-4x^2 + x^2\right) + 2x \\ &= x^4 - 3x^2 + 2x. \end{aligned}$$

(d) $(P(x))^2 - (Q(x))^2$

Use the **difference of squares** identity:
$A^2 - B^2 = (A - B)(A + B)$,
where $A = P(x)$ and $B = Q(x)$.

1. **Compute** $P(x) - Q(x)$:

$$P(x) - Q(x) = (x^2 - 2x + 1) \;-\; (x^2 + 2x) = x^2 - x^2 - 2x - 2x + 1 = -4x + 1.$$

2. **Compute** $P(x) + Q(x)$ (we already have it from part (a)):

$$P(x) + Q(x) = 2x^2 + 1.$$

3. **Multiply** $\left[P(x) - Q(x)\right]\left[P(x) + Q(x)\right]$:

$$(-4x + 1)\,(2x^2 + 1) = -4x \cdot 2x^2 + (-4x) \cdot 1 + 1 \cdot 2x^2 + 1 \cdot 1 = -8x^3 - 4x + 2x^2 + 1.$$

Thus,

$$(P(x))^2 - (Q(x))^2 = \boxed{-8x^3 + 2x^2 - 4x + 1}.$$

3F Division of polynomials

1.

$\frac{x^3+2x^2-5x+6}{x-1}$	Quotient: x^2+3x-2 Remainder: 4
$\frac{2x^3-3x^2+x-2}{x+1}$	Quotient: $2x^2-5x+6$ Remainder: -8
$\frac{3x^3+x^2-4x+1}{x-2}$	Quotient: $3x^2+7x+10$ Remainder: 21
$\frac{x^3-x^2-x+1}{x-1}$	Quotient: x^2-1 Remainder: 0
$\frac{4x^3-12x^2+5x+1}{x-3}$	Quotient: $4x^2+5$ Remainder: 16
$\frac{5x^3+2x^2-3x-14}{x+2}$	Quotient: $5x^2-8x+13$ Remainder: -40
$\frac{x^3-2x^2-9x+18}{x-3}$	Quotient: x^2+x-6 Remainder: 0
$\frac{2x^3-5x^2+4x-7}{x-1}$	Quotient: $2x^2-3x+1$ Remainder: -6
$\frac{3x^3-4x^2-2x+8}{x+1}$	Quotient: $3x^2-7x+5$ Remainder: 3
$\frac{6x^3+x^2-2x-3}{x-2}$	Quotient: $6x^2+13x+24$ Remainder: 45

2.

Provide the remainder of the following polynomial divisions are 0

3G The remainder and factor theorems

1.

(a) $x - 1 \rightarrow -12$

(b) $x + 2 \rightarrow 0$

(c) $x - 3 \rightarrow 50$

(d) $x + 5 \rightarrow -6$

(e) $x \rightarrow -16$

(f) $x - k \rightarrow k^3 + 5k^2 - 2k - 16$

(g) $x + n \rightarrow -n^3 + 5n^2 + 2n - 16$

(h) $x + 3c \rightarrow -27c^3 + 45c^2 + 6c - 16.$

2.

Q1(a): $x^3 - 6x^2 + 11x - 6$ One linear factor: $\boxed{x - 1}$	Q1(b): $x^3 + 3x^2 - 4x - 12$ One linear factor: $\boxed{x - 2}$
Q1(c): $x^3 - 3x^2 - x + 3$ One linear factor: $\boxed{x - 1}$	Q1(d): $2x^3 - 3x^2 - x + 2$ One linear factor: $\boxed{x - 1}$
Q1(e): $x^3 - 2x^2 + x$ One linear factor: $\boxed{x}$ (Also $x - 1$ is a factor of the quadratic part.)	Q1(f): $x^3 + x^2 - 4x - 4$ One linear factor: $\boxed{x - 2}$
Q1(g): $x^3 - 5x^2 + 8x - 4$ One linear factor: $\boxed{x - 1}$	Q1(h): $x^3 + 6x^2 + 11x + 6$ One linear factor: $\boxed{x + 1}$

3.

Q2(1): $x^3 + 2x^2 - x - 2$ $\Longrightarrow (x - 1)(x + 1)(x + 2)$	Q2(2): $x^3 + 2x^2 - 5x - 6$ $\Longrightarrow (x - 2)(x + 1)(x + 3)$
Q2(3): $x^3 - 3x^2 - 4x + 12$ $\Longrightarrow (x - 2)(x - 3)(x + 2)$	Q2(4): $x^3 - 13x + 12$ $\Longrightarrow (x - 1)(x + 4)(x - 3)$
Q2(5): $2x^3 - x^2 - 4x + 3$ $\Longrightarrow (x - 1)^2(2x + 3)$	Q2(6): $2x^3 - 5x^2 + x + 2$ $\Longrightarrow (x - 1)(2x + 1)(x - 2)$
Q2(7): $x^3 - 7x + 6$ $\Longrightarrow (x - 1)(x + 3)(x - 2)$	Q2(8): $x^3 - x^2 - 8x + 12$ $\Longrightarrow (x - 2)^2(x + 3)$
Q2(9): $2x^3 - 4x^2 - 2x + 4$ $\Longrightarrow 2(x - 1)(x - 2)(x + 1)$	Q2(10): $2x^3 - 4x^2 - 10x + 12$ $\Longrightarrow 2(x - 1)(x - 3)(x + 2)$

3H Solving polynomial equations

1.

1(a) $(x+4)(x-1)(x-7)=0 \implies$ $\boxed{x=-4,\ x=1,\ x=7}$	1(b) $(x+4)(x+2)(x-3)=0 \implies$ $\boxed{x=-4,\ x=-2,\ x=3}$
1(c) $(x-6)(x+6)(x-7)=0 \implies$ $\boxed{x=6,\ x=-6,\ x=7}$	1(d) $\left(x+\frac{2}{3}\right)(x-2)\left(x+\frac{2}{5}\right)=0 \implies$ $\boxed{x=-\frac{2}{3},\ x=2,\ x=-\frac{2}{5}}$
1(e) $\left(x+\frac{3}{5}\right)(2x+7)(3x-5)=0 \implies$ $\boxed{x=-\frac{3}{5},\ x=-\frac{7}{2},\ x=\frac{5}{3}}$	1(f) $(7x-4)(17x+3)\left(x-\frac{2}{5}\right)=0 \implies$ $\boxed{x=\frac{4}{7},\ x=-\frac{3}{17},\ x=\frac{2}{5}}$

2.

2(1) $x^3-6x^2+11x-6 \implies$ $\boxed{(x-1)(x-2)(x-3)}$	2(1) $x^3+2x^2-x-2 \implies$ $\boxed{(x-1)(x+1)(x+2)}$
2(2) $2x^3-5x^2+x+2 \implies$ $\boxed{(x-1)(2x^2-3x+2)}$	2(2) $3x^3+6x^2-x-2 \implies$ $\boxed{(x+2)(3x^2-1)}$
2(3) $x^3+x^2-4x-4 \implies$ $\boxed{(x-2)(x+1)(x+2)}$	2(3) $4x^3+8x^2-3x-6 \implies$ $\boxed{(x+2)(4x^2-3)}$
2(4) $x^3-2x^2+x-2 \implies$ $\boxed{(x-2)(x^2+1)}$	2(4) $6x^3-x^2-19x+14 \implies$ $\boxed{(x-1)(6x-7)(x+2)}$
2(5) $x^3+3x^2-4x-12 \implies$ $\boxed{(x-2)(x+2)(x+3)}$	2(5) $x^3-4x^2+5x-2 \implies$ $\boxed{(x-1)^2(x-2)}$
2(6) $x^3-2x^2-41x+42 \implies$ $\boxed{(x-1)(x-7)(x+6)}$	2(6) $2x^3-3x^2-5x+6 \implies$ $\boxed{(x-1)(2x+3)(x-2)}$
2(7) $2x^3-x^2-2x+1 \implies$ $\boxed{(x-1)(2x^2+x-1)=(x-1)(2x-1)(x+1)}$	2(7) $2x^3-3x^2+1 \implies$ $\boxed{(x-1)(2x^2-x-1)=(x-1)^2(2x+1)}$