Chapter 3: Algebra

3A The language of algebra

1.
   a. 3x−2x
   b. x+10
   c. 30−x
   d. x^2
   e. x+8
   f. $\frac{x}{4} + 6$

2.
   a. 5m, −3n, 4p, −6.
   b.
      The coefficient of k is 4.
      The coefficient of m is −3.
      The coefficient of n is 2.
      There is no p-term in the expression, so its coefficient is 0.
   c. 7
   d. 8

3.
   a. True
   b. False
   c. True
   d. True
   e. False
   f. False

4.
   a. y−15
   b. $\frac{y}{5}$
   c. 20

5.
   350 ml cup offers a lower cost per milliliter

6.
   a. 31
   b. 71
   c. Z

3B Substitution and equivalence

1.

10	16	8	8
12	10	13	12
0	5	14/3	13/12

2.

72	96	26	10
21/2	15	36/5	36/7
576	$\frac{36}{\sqrt{10}}$	33/4	10

3.

Expression	$z = 0$	$z = 2$	$z = 5$	$z = 8$
$w = 5z$	0	10	25	40
$w = 2z + 3$	3	7	13	19
$w = 200 - z$	200	198	195	192
$w = z/3$	0	$\frac{2}{3}$	$\frac{5}{3}$	$\frac{8}{3}$
$w = z^2$	0	4	25	64
$w = \frac{150}{z}$	undefined	75	30	18.75
$w = 3(z + 3)$	9	15	24	33
$w = 4z^3$	0	32	500	2048

4.

a. $r = 4\text{ cm},\ h = 6\text{ cm}$

$$V = \pi \times 4^2 \times 6 = \pi \times 16 \times 6 = 96\pi\text{ cm}^3.$$

You can leave it as 96π or approximate numerically.

b. $r = 1.5\text{ m},\ h = 4.5\text{ cm}$

Because the units are mixed (meters for the radius, centimeters for the height), we must convert them to a **single system**. Typically, we choose centimeters:

- $1\text{ m} = 100\text{ cm}$.
- So $r = 1.5\text{ m} = 150\text{ cm}$.

Now $h = 4.5\text{ cm}$ (already in cm). The volume in cm^3 is:

$$V = \pi \times (150)^2 \times 4.5 = \pi \times 22500 \times 4.5 = 101250\,\pi\text{ cm}^3.$$

c. $r = 10\text{ cm},\ h$ is twice the radius

If the height is *twice* the radius, $h = 2r = 2 \times 10 = 20\text{ cm}$. Then

$$V = \pi \times 10^2 \times 20 = \pi \times 100 \times 20 = 2000\pi\text{ cm}^3.$$

d. Diameter = 8 cm, height = (diameter)²

- The diameter is 8 cm, so the radius is $r = 4\text{ cm}$.
- The height is the square of the diameter: $h = (8)^2 = 64\text{ cm}$.

Hence,

$$V = \pi \times 4^2 \times 64 = \pi \times 16 \times 64 = 1024\pi\text{ cm}^3.$$

5.

a. Write an expression for a notebooks and b pens

The total cost C is:

$$C = 4a + 1.50\,b.$$

b. Find the total cost

1. **Six notebooks and four pens**
   Substitute $a = 6,\ b = 4$:
   $$C = 4 \times 6 + 1.50 \times 4 = 24 + 6 = \$30.$$
2. **15 notebooks and 30 pens**
   Substitute $a = 15,\ b = 30$:
   $$C = 4 \times 15 + 1.50 \times 30 = 60 + 45 = \$105.$$

6.

a. $\{\,4x,\ \ 3 + x,\ \ 3x + x\,\}$

- $3x + x = 4x.$
- So $4x$ and $3x + x$ are the same expression.
- The odd one out is $3 + x$.

b. $\{\,2 - a,\ \ a - 2,\ \ a + 1 - 3\,\}$

- $a + 1 - 3 = a - 2.$
- So $a - 2$ and $a + 1 - 3$ are the same expression.
- The odd one out is $2 - a$ (which is -1 times $a - 2$, so it's different).

c. $\{\,5t - 2t,\ \ 2t + t,\ \ 4t - 2t\}$

- $5t - 2t = 3t.$
- $2t + t = 3t.$
- $4t - 2t = 2t.$
- So the first two give $3t$; the third gives $2t$.
- The odd one out is $4t - 2t$.

d. $\{\,8u - 3,\ \ 3u - 8,\ \ 3u - 3 + 5u\}$

- $3u - 3 + 5u = (3u + 5u) - 3 = 8u - 3.$
- So $8u - 3$ and $3u - 3 + 5u$ are the same.
- The odd one out is $3u - 8$.

3C Adding and subtracting terms

1.

- (a) $7m$ & $5m$
- (b) $-4q$ & $\frac{1}{2}q$
- (c) $cd,\ 3dc,\ -2cd$ (all alike)
- (d) $-5t$ & $3t$, then $2s$ & $-4s$, and 3 alone
- (e) $9x$ & $-2x$, then $10xy$ & $3yx$
- (f) $-2kl,\ lk,\ 3lk$ (all alike), $-4k$ alone, $5l$ alone

2.

- (a) $2x + 2y$
- (b) $7z$
- (c) $8a$
- (d) $4 + 4b$
- (e) $4xy - 4y$
- (f) $3x^2 + 6y^2$
- (g) m
- (h) $5r$
- (i) $8s$
- (j) $6p^2$
- (k) $16t^2$
- (l) $9xy - 10t^2$

3.

- (a) $2x - 5$
- (b) $6x^2 - 8x$
- (c) $5z + 9y$
- (d) $m + 5$
- (e) $2xy + 4$
- (f) $4x^2 + 4x - 8$
- (g) $9abc - 3bc$
- (h) $5mn - 3$
- (i) $-2a^2 + 5a - 5$

4.

a) Rectangle $3x, 4x$: $14x$.

b) Square side $6y$: $24y$.

c) Equilateral $\triangle$ side $7z$: $21z$.

d) Parallelogram sides $5a$ and $2a$: $14a$.

e) Trapezium bases $4a, 5b$, slant sides (assuming isosceles) $6a$ each: $4a + 5b + 6a + 6a = 16a + 5b$.

5.

a) $8x + 4y - \boxed{3}x + \boxed{3}y = 5x + 7y$.

b) $3c - 5d + \boxed{4}c + \boxed{3}d = 7c - 2d$.

c) $\boxed{6}m + \boxed{4}n + \boxed{2} = 6m + 4n + 2$.

d) $\boxed{9}p^2q + \boxed{5}pq^2 + \boxed{3}q^2p$.

6.

a) Substitution shows $5x + 4y \neq 9xy$ in general (e.g. $x = 2, y = 1$ yields $5(2) + 4(1) = 14 \neq 18$).

b) Yes, $3c + 2d = 6cd$ can be solved; e.g. $c = 1, d = \frac{3}{4}$.

c) $5p + 5q \neq 10p$ unless $q = p$. In general they differ.

7.

$15a - 5b + 6a + 2b = 21a - 3b$.

3D Multiplying and dividing terms

1.

a) $5x \times 2 = 10x$

b) $3y \times 4 = 12y$

c) $7m \times 2 = 14m$

d) $6x \times 5y = 30xy$

e) $8x \times 2y = 16xy$

f) $10x \times 4y = 40xy$

g) $5xy \times 3x = 15x^2y$

h) $7x \times 2y \times 3 = 42xy$

i) $3xy \times 2xy = 6x^2y^2$

j) $5y \times 4x \times 6y = (5 \times 4 \times 6)\,(x)\,(y \cdot y) = 120\,x\,y^2$

k) $3a \times 5ab = 15a^2b$

l) $4ab \times 3bc = 12ab^2c$

m) $9abc \times 3a = 27a^2bc$

n) $2 \times 4abc \times 5 = 40abc$

o) $15x^2 \times 3 \times 2y^2 = 90x^2y^2$

2.

a) $24x^2$, b) $30xy$, c) $24xz^2$, d) $2x^3yz$, e) $24x^2$, f) $36xy^2$, g) $8xyz$, h) $12a^2b^2$, i) $30xy$, j) $12x^2y$, k) $6x^3y^2$, l) $84x^2y^2z^2$, m) $12x^3y^3$, n) $80x^3y$, o) $24x^3y^2z$, p) $12x^5y^2$, q) $60x^4y^2$, r) $24x^3y^2$.

3.

a) $3y$, b) $10x$, c) $2x$, d) $4y$, e) $4x$, f) $2y$, g) $\frac{1}{2}y$, h) $\frac{1}{3y}$, i) $\frac{1}{2}z$, j) $3y$, k) $\frac{y}{8x}$, l) $\frac{1}{12}$.

4.

a) $2x$, b) $6y$, c) $\frac{4x}{y}$, d) 5, e) $7\frac{x^2}{y^2}$, f) $3x$, g) $\frac{1}{3}x$, h) 3, i) $9y$, j) $3y$, k) $4xy$, l) $\frac{4y}{x}$.

5.

a) $\frac{3xy}{8}$, b) $\frac{xy}{15}$, c) $\frac{6x^2y}{35}$, d) $\frac{8x}{3y}$, e) $\frac{xy}{9}$, f) $\frac{35x^2}{12}$, g) 3, h) xy.

6.

a) $3bd$

b) $\frac{4}{3}a^2$

c) $\frac{8x-3y}{16}$

d) $2a$

e) 1

f) 0

7.

a) Rectangle ($4y \times 9y$): $36y^2$.

b) Triangle ($\frac{1}{2} \times 7b \times 6c$): $21bc$.

c) Circle ($\pi(2r)^2$): $4\pi r^2$.

3E Adding and subtracting algebraic fractions

1.

1a) $\frac{3x}{4} + \frac{5x}{4} = 2x$

1b) $\frac{4a}{3} + \frac{a}{3} = \frac{5a}{3}$

1c) $\frac{b}{6} + \frac{3b}{6} = \frac{4b}{6} = \frac{2b}{3}$

1d) $\frac{5k}{2} + \frac{2k}{2} = \frac{7k}{2}$

1e) $\frac{d}{5} + \frac{4d}{5} = d$

1f) $\frac{c}{2} + \frac{c}{3} = \frac{5c}{6}$

1g) $\frac{m}{3} + \frac{2m}{4} = \frac{5m}{6}$

1h) $\frac{7p}{9} + \frac{4p}{9} = \frac{11p}{9}$

1i) $\frac{x}{6} + \frac{5x}{12} = \frac{7x}{12}$

1j) $\frac{3y}{5} - \frac{2y}{5} = \frac{y}{5}$

1k) $\frac{5r}{8} - \frac{2r}{8} = \frac{3r}{8}$

1l) $\frac{t}{11} - \frac{3t}{2} = -\frac{31t}{22}$

1m) $\frac{6w}{5} - \frac{2w}{8} = \frac{19w}{20}$

1n) $\frac{2u}{3} - \frac{u}{4} = \frac{5u}{12}$

1o) $\frac{h}{7} - \frac{3h}{6} = -\frac{5h}{14}$

1p) $\frac{3y}{11} - \frac{6c}{12}$ (cannot combine y and c; final form)

2.

2a) $5x + \frac{x}{4} = \frac{21x}{4}$

2b) $2x + \frac{x}{6} = \frac{13x}{6}$

2c) $\frac{b}{4} + 3b = \frac{13b}{4}$

2d) $\frac{9p}{2} - 4p = \frac{p}{2}$

2e) $\frac{u}{5} + 6u = \frac{31u}{5}$

2f) $\frac{3m}{7} + \frac{5v}{9}$ (different variables; no further simplification)

2g) $t + \frac{6p}{3} = t + 2p$

2h) $\frac{k}{5} - m$, or $\frac{k-5m}{5}$ (both forms acceptable)

3.

3a) $\frac{m}{5} + \frac{m}{4} + \frac{m}{20} = \frac{m}{2}$

3b) $\frac{3x}{7} + \frac{2x}{3} - \frac{x}{7} = \frac{20x}{21}$

3c) $\frac{5b}{3} + \frac{2b}{5} - \frac{4b}{6} = \frac{7b}{5}$

3d) $\frac{9p}{4} + \frac{p}{6} - \frac{2p}{3} = \frac{7p}{4}$

3e) $\frac{c}{2} + \frac{c}{3} - 4 = \frac{5c-24}{6}$

3f) $\frac{d}{5} + \frac{d}{7} + \frac{d}{10} = \frac{31d}{70}$

3g) $\frac{6j}{8} - \frac{2j}{5} + 3 = \frac{7j}{20} + 3$

3h) $\frac{2t}{3} - \frac{4t}{5} + \frac{t}{6} = \frac{t}{30}$

4.

a) Total as sum: $\frac{1}{2}J + \frac{1}{3}C$

b) Single fraction: $\frac{3J+2C}{6}$

c) If $J = 1500$ liters, $C = 3$ liters:

$$\frac{1}{2} \times 1500 + \frac{1}{3} \times 3 = 750 + 1 = 751 \text{ liters.}$$

5.

a) Savings: $\frac{3}{8}M$

b) Leisure: $\frac{2}{5}M$

c) Total (savings + leisure):

$$\frac{3}{8}M + \frac{2}{5}M = \frac{31}{40}M.$$

6.

Initial balance B.

Reduced by one-third → new balance = $\frac{2}{3}B$.

Then $\$15$ withdrawn:

$$\text{Final amount} = \frac{2}{3}B - 15.$$

7.

a) $\frac{y}{3} + \frac{y}{4} \equiv \frac{7y}{12}$. Substitutions confirm they match.

b) $\frac{z}{5} + \frac{z}{6}$ is not $\frac{11z}{25}$. Actually $\frac{11z}{30} \neq \frac{11z}{25}$.

c) $\frac{m}{4} + \frac{m}{6} = \frac{5m}{12}$, while $m - \frac{m}{12} = \frac{11m}{12}$. So they differ, not equivalent.

3F Multiplying and dividing algebraic fractions

1.

(a) $\frac{5a}{4} \times \frac{6b}{7}$ $\frac{15ab}{14}$	(b) $\frac{3x}{2} \times \frac{4y}{5}$ $\frac{6xy}{5}$	(c) $\frac{7d}{5} \times \frac{9c}{8}$ $\frac{63dc}{40}$	(d) $\frac{8m}{3} \times \frac{2n}{5}$ $\frac{16mn}{15}$
(e) $\frac{6p}{7} \times \frac{3q}{2}$ $\frac{9pq}{7}$	(f) $\frac{2z}{5} \times \frac{5k}{4}$ $\frac{1}{2}zk$	(g) $\frac{4b}{7} \div \frac{2c}{5}$ $\frac{10b}{7c}$	(h) $\frac{3x}{4} \div \frac{5y}{8}$ $\frac{6x}{5y}$
(i) $\frac{7p}{6} \div \frac{4m}{9}$ $\frac{21p}{8m}$	(j) $\frac{9y}{5} \div \frac{5z}{3}$ $\frac{27y}{25z}$	(k) $\frac{2k}{3} \times \frac{7m}{5}$ $\frac{14km}{15}$	(l) $\frac{3a}{8} \div \frac{5b}{2}$ $\frac{3a}{20b}$
(m) $\frac{5c}{6} \times \frac{4d}{3}$ $\frac{10cd}{9}$	(n) $\frac{6p}{7} \div \frac{9q}{5}$ $\frac{10p}{21q}$	(o) $\frac{4y}{5} \div \frac{3x}{2}$ $\frac{8y}{15x}$	(p) $\frac{8b}{9} \times \frac{7c}{2}$ $\frac{28bc}{9}$
(q) $\frac{3u}{4} \div \frac{5v}{2}$ $\frac{3u}{10v}$	(r) $\frac{6m}{5} \times \frac{3n}{7}$ $\frac{18mn}{35}$	(s) $\frac{5x}{3} \div \frac{4y}{6}$ $\frac{5x}{2y}$	(t) $2 \div \frac{3x}{5}$ $\frac{10}{3x}$

2.

(a) $\frac{4a}{b} \times \frac{2a}{3b}$ $\frac{8a^2}{3b^2}$	(b) $\frac{6x}{y} \times \frac{5x}{2y}$ $\frac{15x^2}{y^2}$	(c) $m \times \frac{4}{m^2}$ $\frac{4}{m}$	(d) $\frac{2pq}{3r} \times \frac{5rs}{2p}$ $\frac{5qs}{3}$
(e) $\frac{-8bc}{12abc} \times \frac{3a}{4c}$ $\frac{-1}{2c}$	(f) $\frac{7t}{mn} \div \frac{2}{n}$ $\frac{7t}{2m}$	(g) $\frac{-5jk}{6p} \div \frac{3kp}{8j}$ $\frac{-20j^2}{9p^2}$	(h) $\frac{8xw}{9wz} \div \frac{4x}{12z}$ $\frac{8}{3}$
(i) $\frac{9mn}{5op} \div \frac{3o}{pn}$ $\frac{3m\,n^2}{5o^2}$	(j) $\frac{-7g}{5h} \div \frac{3k}{4gh}$ $\frac{-28g^2}{15k}$	(k) $\frac{20ab}{6} \times \frac{12bc}{7a}$ $\frac{40b^2c}{7}$	(l) $\frac{3cv}{5f} \times \frac{10f}{8d}$ $\frac{3cv}{4d}$
(m) $\frac{9p}{q} \div \frac{2}{pq}$ $\frac{9p^2}{2}$	(n) $\frac{-4gh}{2j} \times \frac{8j}{9gh}$ $\frac{-16}{9}$	(o) $\frac{15xt}{25y} \div \frac{10at}{5xy}$ $\frac{3x^2}{10a}$	(p) $\frac{18bc}{12} \times \frac{9cd}{6b}$ $\frac{9c^2d}{4}$

3.

a) Kevin's payment: $\frac{y}{2}$

b) Additional coverage (one-fourth of Mike's half): $\frac{y}{8}$

4.

a) Original area: $\frac{bh}{2}$

b) Both base & height halved → new area: $\frac{bh}{8}$

c) Base $\to \frac{5b}{8}$, height $= h$. New area $\frac{5bh}{16}$ (which is 0.625 of the original area $\frac{bh}{2}$; less than full, more than half)

d) Base $\to 0.7b$, height $\to 0.7h$. New area: $\frac{1}{2}(0.7b)(0.7h) = 0.49 \cdot \frac{bh}{2}$

5.

a) "p doubled, then divided by 4": $\frac{p}{2}$

b) "y multiplied by $\frac{3}{5}$, then divide by $\frac{1}{4}$": $\frac{12y}{5}$

c) "$\frac{m}{n}$ divided by its reciprocal $\frac{n}{m}$": $\frac{m^2}{n^2}$

d) "z increased by 10%, then tripled": $3.3z$ or $\frac{33z}{10}$

3G Expanding brackets

1.

a) $6x - 12$
b) $20y + 24$
c) $24z - 18$
d) $15p - 35$
e) $-16x - 10$
f) $35x - 14$
g) $-18y + 12$
h) $-45p - 10$
i) $12x + 20$
j) $-12z + 18$
k) $-21p + 12$
l) $10x + 2$
m) $15x - 20y$
n) $16x - 8y$
o) $-24y + 20x$
p) $27p + 18x$
q) $35z - 7y$
r) $10x + 40$
s) $-21x + 35$
t) $12p + 6y$
u) $-20x + 15y$
v) $14z + 10$
w) $-16p + 48$
x) $15x + 27y$

2.

a) $3x^2 - 3xy$
b) $4xy + 4y^2$
c) $10x^2 + 15xy$
d) $8x^2 - 2xy$
e) $3xy - 2y^2$
f) $8xy + 20y$
g) $15xy - 6xy^2$
h) $8x^2 - 6x^3$
i) $15x^2 - 5x^3$
j) $28x - 12xy$
k) $30y - 6xy$
l) $18x - 3xy$
m) $6x^2y - 2x^3y$
n) $8xy^2 - 4x^2y^2$
o) $12xy^2 + 3xy^3$
p) $7x^2y - x^3y$
q) $40x^2 - 24x^3$
r) $20xy^2 - 10x^2y^2$

3.

a) 144
b) 14
c) 6.5
d) 60
e) 120
f) -2400
g) 12
h) 54
i) 81
j) 82
k) 22.5
l) -88.8

4.

a) $2m + 3i$
b) $200m + 300i$
c) $225m + 325i$
d) For $m = 20, i = 5$, total cost $= 6125$

5.

a) $\frac{5x+9}{12}$
b) $\frac{9x+26}{20}$
c) $\frac{5x-6}{9}$
d) $\frac{11x+26}{30}$
e) $\frac{38x+23}{56}$
f) $\frac{28x+9}{30}$

3H Factorising expressions

1.

(a) $4x + 8 = 4(x + 2)$, (b) $3y + 9 = 3(y + 3)$, (c) $6g + 12 = 6(g + 2)$,
(d) $10x + 15 = 5(2x + 3)$, (e) $5f + 10 = 5(f + 2)$, (f) $14c + 28 = 14(c + 2)$,
(g) $2h + 6 = 2(h + 3)$, (h) $3x - 6 = 3(x - 2)$, (i) $8g - 16 = 8(g - 2)$,
(j) $9k + 18 = 9(k + 2)$, (k) $6s - 12 = 6(s - 2)$, (l) $5x - 10 = 5(x - 2)$,
(m) $20g - 40 = 20(g - 2)$, (n) $12a - 24 = 12(a - 2)$, (o) $8b + 16 = 8(b + 2)$,
(p) $30x + 60 = 30(x + 2)$, (q) $9z - 18 = 9(z - 2)$, (r) $7d - 14 = 7(d - 2)$,
(s) $16x + 32 = 16(x + 2)$, (t) $4v + 8 = 4(v + 2)$, (u) $6p + 12 = 6(p + 2)$,
(v) $18z - 36 = 18(z - 2)$, (w) $10w + 20 = 10(w + 2)$, (x) $6j - 12 = 6(j - 2)$.

2.

(a) $4gh + 16 = 4(gh + 4)$, (b) $3xy + 9y = 3y(x + 3)$, (c) $15pq + 5p = 5p(3q + 1)$,
(d) $18g - 9gh = 9g(2 - h)$, (e) $20jk - 4k = 4k(5j - 1)$, (f) $10eg + 5g = 5g(2e + 1)$,
(g) $14k + 28 = 14(k + 2)$, (h) $8mn + 6m = 2m(4n + 3)$, (i) $16ab + 8b = 8b(2a + 1)$,
(j) $6a - 18abc = 6a(1 - 3bc)$, (k) $9r + 15rt = 3r(3 + 5t)$, (l) $20mab + 10ab = 10ab(2m + 1)$,
(m) $14cn + 28n = 14n(c + 2)$, (n) $30y + 10ry = 10y(3 + r)$, (o) $18jn + 12n = 6n(3j + 2)$,
(p) $28g + 24gj = 4g(7 + 6j)$, (q) $12h + 6z = 6(2h + z)$, (r) $36u - 24n = 12(3u - 2n)$,
(s) $45y + 63ay = 9y(5 + 7a)$, (t) $14d + 21dz = 7d(2 + 3z)$, (u) $27hm - 12mx = 3m(9h - 4x)$.

3.

(a) $\frac{(5x-5)}{(15x-30)} \times \frac{(12x+24)}{(6x-6)} \times \frac{(4x-8)}{(16x+32)} = \boxed{\frac{1}{6}}$.

(b) $\frac{(6x-9)}{(18x-27)} \times \frac{(8x+16)}{(4x-4)} \times \frac{(10x-20)}{(20x+40)} = \boxed{\frac{x-2}{3(x-1)}}$.

4.

$\boxed{\frac{15\,(a+2b)}{2\,(b+3a)}}$.

5.

$\boxed{\frac{y(7ax-10b)}{ax-2b}}$.

6.

1. $\dfrac{3x+6}{9x+18} = \frac{1}{3}$.
2. $\dfrac{8y-8}{4y-4} = 2$.
3. $\dfrac{2ac+4a}{6a+12ab} = \frac{c+2}{3(1+2b)}$.
4. $\dfrac{5a+10b}{10a+20b} = \frac{1}{2}$.
5. $\dfrac{9q-18}{3q-6} = 3$.
6. $\dfrac{6p+12pq}{18p+36pq} = \frac{1}{3}$.
7. $\dfrac{5a-15}{a-3} = 5$.
8. $\dfrac{16p}{4p+2pq} = \dfrac{8}{2+q}$.
9. $\dfrac{50-5x}{25-5x} = \dfrac{x-10}{x-5}$.
10. $\dfrac{4x+8}{8x+16} = \frac{1}{2}$.

7.

$$\text{width} = \frac{12b+9}{4b+3} = \frac{3(4b+3)}{4b+3} = \boxed{3}.$$

3I Applying algebra

1.

$$5x + 8 = 2x - 3$$

Simplify:

$$5x - 2x = -3 - 8 \implies 3x = -11 \implies x = -\frac{11}{3}$$

2.

$$2x - 7 = \frac{x}{6} + 11$$

Simplify:

$$2x - \frac{x}{6} = 18 \implies \frac{12x - x}{6} = 18 \implies \frac{11x}{6} = 18$$

$$11x = 108 \implies x = \frac{108}{11}$$

3.

$$\frac{x-12}{9} = 3x + 26$$

Simplify:

$$x - 12 = 9(3x + 26) \implies x - 12 = 27x + 234$$

$$x - 27x = 234 + 12 \implies -26x = 246 \implies x = -\frac{246}{26} = -\frac{123}{13}$$

4.

Plan 1: $40 + 3(s-5)$ (if $s > 5$)
Plan 2: $20 + 5s$

Set them equal:

$$40 + 3s - 15 = 20 + 5s \implies 25 + 3s = 20 + 5s$$

$$25 - 20 = 5s - 3s \implies 5 = 2s \implies s = 2.5$$

Answer: The plans cost the same after **2.5 sessions**.

5.

Let a be the cost of an adult ticket and $c = \frac{1}{3}a$ be the cost of a child ticket.

$$2a + 4c = 40 \implies 2a + 4\left(\frac{1}{3}a\right) = 40$$

Simplify:

$$2a + \frac{4a}{3} = 40 \implies \frac{6a + 4a}{3} = 40 \implies \frac{10a}{3} = 40$$

$$10a = 120 \implies a = 12$$

$$c = \frac{1}{3} \times 12 = 4$$

6.

The average of six numbers is 45:

$$\frac{\text{Sum of six numbers}}{6} = 45 \implies \text{Sum} = 270$$

After adding a seventh number x, the average becomes 50:

$$\frac{270 + x}{7} = 50 \implies 270 + x = 350 \implies x = 80$$

Answer: The seventh number is **80**.

7.

The current total score:

$$68 + 72 + 85 + 90 + 77 = 392$$

Let x be the next score. To average 80 after 6 quizzes:

$$\frac{392 + x}{6} = 80 \implies 392 + x = 480 \implies x = 88$$

Answer: Sarah needs a score of **88**.

8.

After 5 assignments, the total score is:

$$\frac{\text{Total}}{5} = 82 \implies \text{Total} = 410$$

After the 6th assignment, the average is 85:

$$\frac{410 + x}{6} = 85 \implies 410 + x = 510 \implies x = 100$$

Answer: The 6th assignment score is **100**.

9.

Let the medium coffee cost m. Then:

$$\text{Small} = m - 2, \quad \text{Large} = 1.5m$$

$$(m - 2) + m + 1.5m = 14.50$$

Simplify:

$$3.5m - 2 = 14.50 \implies 3.5m = 16.50 \implies m = \frac{16.50}{3.5} = 4.71$$

Answer: Medium coffee costs **$4.71**.

10.

Let x be the cost of chips. Then:

$$\text{Water} = x + 0.50, \quad \text{Chocolate} = x + 1.50$$

$$x + (x + 0.50) + (x + 1.50) = 5$$

Simplify:

$$3x + 2 = 5 \implies 3x = 3 \implies x = 1$$

$$\text{Chips} = 1, \quad \text{Water} = 1.50, \quad \text{Chocolate} = 2.50$$

Answer: Chips: $1, Water: $1.50, Chocolate: $2.50.

11.

Cost formula: $3 + 1.5x$

a.

i. Alice: $2(3 + x)$

ii. Ben: $2x + 3$

iii. Cara: $4\left(\frac{x}{2} + 2\right)$

b. Fourth rider's calculation: Multiply miles by 5, add 6:

$$5x + 6$$

To get the correct fare, subtract $2x - 3$.

12.

- For 1 hour: $P + R$
- For 2 hours: $P + 2R$
- For 45 minutes ($t = 0.75$): $P + 0.75R$
- For t hours: $P + tR$

3J Index laws Multiplying and dividing powers

1.

a. x^7
b. $7y^9$
c. $12a^9$
d. z^8
e. $-10x^6$
f. $-12y^7$
g. $2b^{11}$
h. $18a^5$
i. $7y^9$
j. $15x^8$
k. $-24b^{11}$
l. $8x^{10}$
m. z^6
n. a^9b^5
o. $-3b^{16}$
p. $-4a^8$

2.

a. $a^3 \times a^4 = a^7$
b. $5x^9 \times 5x^3 = 25x^{12}$
c. $c^{11} \times c^{-5} = c^6$
d. $3x^8 \times 5x^6 = 15x^{14}$

3.

a. $18m^6$
b. $8k^{11}$
c. $27x^{11}$
d. $40y^{13}$
e. m^5n^{11}
f. x^5y^5
g. r^4s^9
h. $y^{15}z^5$
i. $135x^3$
j. $8a^5$
k. $15x^5y^9$
l. $18a^5b^3$
m. $-20x^8y^5$
n. $-6a^3b^4$
o. $-6c^7d$
p. $45x^5y^3$

4.

a. x^4
b. $6y^7$
c. $\frac{1}{7}m^6$
d. b^5
e. p^7
f. x^4
g. r^3
h. s^7
i. $\frac{5}{3}k^5$
j. $\frac{7}{2}g^7$
k. $2p^6$
l. $3l^5$

5.

a. $a^{10} \div a^4 = a^6$
b. $x^7 \div x^4 = x^3$
c. $p^5 \div p^{-4} = p^9$
d. $k^{10} \div k^6 = k^4$

6.

a. m^3
b. z^4
c. q^4
d. r^9
e. p^5
f. n^5
g. a^5
h. k^8
i. $4m^5$
j. $7y^5$
k. x^{10}
l. $5b^4$
m. x^4y^6
n. p^3q^4
o. $3a^5$
p. b^6

7.

a.

i. 9
ii. -27
iii. 81
iv. -243

b. $(-3)^6 = 729$

8.

a. $a \div b = 16$
b. $4a \div 4b = 27$

9.

$a = 2, b = 4$

3K Index laws Raising powers

1.

a. x^6
b. y^{20}
c. z^{12}
d. u^{72}
e. $16x^{12}$
f. $9x^4y^8$
g. 1
h. $64y^9$
i. x^8y^{20}
j. $x^{20}y^{15}$
k. $8a^{15}b^{18}$
l. x^2y^8
m. x^{15}
n. y^{12}
o. x^9
p. 1

2.

a. x^{18}
b. y^{27}
c. $200p^{13}$
d. $64m^{12}$
e. $135a^{20}$
f. $400k^{16}$
g. $432r^{14}$
h. $400p^{10}$
i. z^{13}
j. p^{11}
k. $\frac{81}{x^3}$
l. $\frac{1}{y^2}$
m. $8m^2$
n. $\frac{1}{q^3}$
o. $\frac{125}{9p^2}$
p. m^8k^7

3.

a. $6x^6$
b. $15x^4y$
c. $2x^5$
d. $\frac{x^4}{3}$
e. $35x^4a^6b^2$
f. $3x^4 + 24x$
g. $5x^3 - x^4$
h. 1
i. $\frac{6}{x^5y^2}$
j. $\frac{6x^4}{5}$
k. $\frac{x^{15}}{y^6}$
l. $\frac{xy^9}{16}$
m. 1
n. $10x^6$
o. $\frac{9y^3}{x^5}$

4.

a. $\frac{1}{3x^2}$
b. x^{10}
c. y^3z^5
d. $\frac{1}{a^2b^4c^5}$
e. $x^{14}y^0 = x^{14}$
f. $\frac{1}{9^{10}}$
g. 5^{10}
h. 7^9
i. 10^6

5.

a. $x = 4$
b. $a = 6$
c. $b = 3, c = 3$

6.

a. 16
b. 4
c. 4
d. 5

7.

i. **3 zeros**
ii. **4 zeros**
iii. **8 zeros**

b. **20 zeros**