Homework Solution

4A Trigonometric ratios

1.

Expression	Approx. Value
a) $\sin 15^\circ$	0.2588
b) $\cos 20^\circ$	0.9397
c) $\tan 25^\circ$	0.4669
d) $\sin 30^\circ$	0.5000
e) $ an 70^\circ$	2.7475
f) $\cos 42^\circ$	0.7431
g) $\cos 170^\circ$	-0.9848
h) $ an 68^\circ$	2.4751
i) $\tan 72^\circ$	3.0777
j) $\sin 49^{\circ}$	0.7547
k) $\sin 53^\circ$	0.7980
I) $\cos 86^{\circ}$	0.0698
m) $\cos 57^{\circ}$	0.5446
n) $ an 120^\circ$	-1.7321
o) $ an 66^\circ$	2.2460
p) $\cos 103^{\circ}$	-0.2225

2.

$$\sin \alpha = \frac{a}{c}; \sin \beta = \frac{b}{c}$$
$$\cos \alpha = \frac{b}{c}; \cos \beta = \frac{a}{c}$$
$$\tan \alpha = \frac{a}{b}; \tan \beta = \frac{b}{a}$$

3.

X=16.98; y=14.40	X=8.62; y=21.33
X=15.69; y=4.59	X=26.91; y=18.00

4.

a. 2.25m

b. 3.9m

5.

a. 6.4m

b. 2.8m

6.

a. 309m

b. 951m

4B Finding unknown angles

1.

$\alpha = 53.13; \beta = 36.87$	$\alpha = 36.84; \beta = 53.13$	$\alpha = 40.71; \beta = 49.29$
$\alpha = 41.01; \beta = 48.99$	$\alpha = 41.989; \beta = 48.01$	$\alpha = 33.06; \beta = 56.94$
$\alpha = 39.32; \beta = 50.68$	$\alpha = 40.18; \beta = 49.82$	$\alpha = 34.53; \beta = 55.47$
$\alpha = 37.30; \beta = 52.70$	$\alpha = 42.97; \beta = 47.03$	$\alpha = 38.86; \beta = 51.14$

2.

70.82°

3.

62.4°

4.

10.06°

5.

8.53°

4C Applications using angles of elevation and depression

1. Lighthouse and Boat:

(b) The top of the lighthouse is approximately **149.89 m** above the water.

(c) The cliff is approximately **119.89 m** high.

2. Tower and Tourist:

(a) The observation deck is about **15 m** above the ground.

(b) The top of the tower is about **45 m** above the observation deck.

3. Airplane Observations:

(a) The plane is approximately **6436 m** horizontally from the first person.

(b) The second person calculates the plane's altitude as about 932 m.

4. **Observation from a Tower:**

(a) The car is approximately 213.4 m from the base of the tower.(b) The helicopter is about 75.35 m above the ground.

5. Lamp Post and Building:

(a) The building is about **11.19 m** taller than the lamp post.

(b) The total height of the building is approximately **14.69 m**.

6. Hot Air Balloon and Car:

(a) At position A, the angle of elevation is about **0.91**°.

(b) At position B, the angle of elevation is about 2.15° .

(c) The difference between the angles is approximately 1.24° .

4D Bearings

1. $N30^{\circ}E \rightarrow 030^{\circ}$ $N45^{\circ}W \rightarrow 315^{\circ}$ $S12^{\circ}E \rightarrow 168^{\circ}$ $S15^{\circ}W \rightarrow 195^{\circ}$ $N60^{\circ}E \rightarrow 060^{\circ}$ $S42^{\circ}W \rightarrow 222^{\circ}$ $S75^{\circ}E \rightarrow 105^{\circ}$ $N88^{\circ}W \rightarrow 272^{\circ}$

2.

 $030^{\circ}T \rightarrow N30^{\circ}E$ $097^{\circ}T \rightarrow E7^{\circ}S$ $103^{\circ}T \rightarrow E13^{\circ}S$ $118^{\circ}T \rightarrow E28^{\circ}S$ $220^{\circ}T \rightarrow S40^{\circ}W$ $242^{\circ}T \rightarrow S62^{\circ}W$ $276^{\circ}T \rightarrow N84^{\circ}W$ $350^{\circ}T \rightarrow N10^{\circ}W$

- A: 035°T
- B: 127°T
- C: 225°T
- D: 332°T
- E: 055°T
- F: 137°T
- G: 210°T
- H: 298°T
- 4. 3.98 km
- 5. 3.43 km westward; 2.34 km southward
- 6. 5.12 km northward; 5.47 km eastward

4E Applications in three dimensions

1.

30.80°

2.

43.27 cm

3.

4044.3951 *cm*²

4.

18.66 m

4F The sine rule

1.

10.4	16.0	22.9
39.2	21.9	15.3

2.

40.7	45.1	71.9
50.1	49.1	33.4

4G The cosine rule

1.

43.51	17.20
24.22	21.58
8.6	63.36

2.

60	65.14	20.66

3. 72°

- 4. 1586 m
- 5. 26.7 cm

4H Area of a triangle

1.

- a. 20
- b. 12.32
- c. 54.56
- d. 213.99
- e. 3.55
- f. 42.27
- g. 41.21
- h. 96.29
- i. 52.6
- j. 2.91

2.

74.83 cm^2	149.5 cm^2
304.94 <i>cm</i> ²	29.77 cm^2
18.97 <i>cm</i> ²	$38.68 \ cm^2$
45.17 cm^2	52.31 <i>cm</i> ²

4I The unit circle

- a) 37° Quadrant I
- b) 158° Quadrant II c) 105° – Quadrant II
- d) 193° Quadrant III
- e) 281° Quadrant III
- f) 273° Quadrant IV
- g) 352° Quadrant IV
- h) 720° Quadrant I
- i) 793° Quadrant I
- j) 812° Quadrant II
- k) 863° Quadrant II
- 1) 885° Quadrant II

2.

170°	157°	145°
114°	168°	123°
166°	108°	135°
167°	175°	132°

3.

a) 45°: Quadrant I; sin is positive, cos is positive, tan is positive
b) 87°: Quadrant I; sin is positive, cos is positive, tan is positive
c) 152°: Quadrant II; sin is positive, cos is negative, tan is negative
d) 138°: Quadrant II; sin is positive, cos is negative, tan is negative
e) 182°: Quadrant III; sin is negative, cos is negative, tan is positive
f) 193°: Quadrant III; sin is negative, cos is negative, tan is positive
g) 210°: Quadrant III; sin is negative, cos is negative, tan is positive
g) 210°: Quadrant III; sin is negative, cos is negative, tan is positive
h) 256°: Quadrant III; sin is negative, cos is negative, tan is positive
i) 287°: Quadrant IV; sin is negative, cos is positive, tan is negative
j) 305°: Quadrant IV; sin is negative, cos is positive, tan is negative
k) 331°: Quadrant IV; sin is negative, cos is positive, tan is negative
l) 379° (or 19° coterminal): Quadrant I; sin is positive, cos is positive, tan is positive

4.

a) $\sin 125^{\circ} = \sin 55^{\circ}$ b) $\cos 213^{\circ} = -\cos 33^{\circ}$ c) $\tan 195^{\circ} = \tan 15^{\circ}$ d) $\sin 123^{\circ} = \sin 57^{\circ}$ e) $\cos 298^{\circ} = \cos 62^{\circ}$ f) $\tan 352^{\circ} = -\tan 8^{\circ}$ g) $\sin 158^{\circ} = \sin 22^{\circ}$ h) $\cos 236^{\circ} = -\cos 56^{\circ}$ i) $\tan 248^{\circ} = \tan 68^{\circ}$ j) $\sin 178^{\circ} = \sin 2^{\circ}$ k) $\cos 325^{\circ} = \cos 35^{\circ}$ l) $\tan 245^{\circ} = \tan 65^{\circ}$

4J Exact values and radians

1.

a.
$$50^{\circ} = \frac{5\pi}{18}$$

b. $45^{\circ} = \frac{\pi}{4}$
c. $80^{\circ} = \frac{4\pi}{9}$
d. $140^{\circ} = \frac{7\pi}{9}$
e. $180^{\circ} = \pi$
f. $200^{\circ} = \frac{10\pi}{9}$
g. $300^{\circ} = \frac{5\pi}{3}$
h. $360^{\circ} = 2\pi$
i. $10^{\circ} = \frac{\pi}{18}$
j. $250^{\circ} = \frac{25\pi}{18}$
k. $150^{\circ} = \frac{5\pi}{6}$
l. $2^{\circ} = \frac{\pi}{90}$

a.
$$1.2\pi = 216^{\circ}$$

b. $\frac{3\pi}{8} = 67.5^{\circ}$
c. $\frac{\pi}{6} = 30^{\circ}$
d. $\frac{2\pi}{9} = 40^{\circ}$
e. $\frac{7\pi}{18} = 70^{\circ}$
f. $\frac{11\pi}{12} = 165^{\circ}$
g. $\frac{4\pi}{5} = 144^{\circ}$
h. $\frac{5\pi}{8} = 112.5^{\circ}$
i. $\frac{2\pi}{3} = 120^{\circ}$
j. $\frac{13\pi}{12} = 195^{\circ}$
k. $2.5\pi = 450^{\circ}$
l. $1.4\pi = 252^{\circ}$

a)
$$\cos 30^{\circ} = \frac{\sqrt{3}}{2}$$

b) $\sin 45^{\circ} = \frac{\sqrt{2}}{2}$
c) $\tan 60^{\circ} = \sqrt{3}$
d) $\cos 45^{\circ} = \frac{\sqrt{2}}{2}$
e) $\cos 150^{\circ} = -\frac{\sqrt{3}}{2}$ (since 150° has a 30° reference angle in QII)
f) $\tan 120^{\circ} = -\sqrt{3}$ (120° in QII, reference angle 60°)
g) $\sin 135^{\circ} = \frac{\sqrt{2}}{2}$ (135° in QII, reference angle 45°)
h) $\cos 135^{\circ} = -\frac{\sqrt{2}}{2}$
i) $\sin 120^{\circ} = \frac{\sqrt{3}}{2}$ (120° in QII, reference angle 60°)
j) $\tan 150^{\circ} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ (150° in QII, reference angle 30°)
k) $\cos 120^{\circ} = -\frac{1}{2}$ (120° in QII, reference angle 60°)
l) $\sin 150^{\circ} = \frac{1}{2}$ (150° in QII, reference angle 60°)
m) $\tan 135^{\circ} = -1$ (135° in QII, reference angle 30°)
m) $\tan 135^{\circ} = -1$ (135° in QII, reference angle 45°)
n) $\sin 90^{\circ} = 1$
o) $\cos 90^{\circ} = 0$
p) $\tan 90^{\circ}$ is undefined

a)
$$\cos 210^{\circ} = -\frac{\sqrt{3}}{2}$$
 (210° in QIII, reference angle 30°)
b) $\sin 225^{\circ} = -\frac{\sqrt{2}}{2}$ (225° in QIII, reference angle 45°)
c) $\tan 240^{\circ} = \sqrt{3}$ (240° in QIII, reference angle 60°; tangent positive in QIII)
d) $\cos 300^{\circ} = \frac{1}{2}$ (300° in QIV, reference angle 60°)
e) $\sin 315^{\circ} = -\frac{\sqrt{2}}{2}$ (315° in QIV, reference angle 45°)
f) $\tan 210^{\circ} = \tan 30^{\circ} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (210° in QIII, reference angle 30°)
g) $\cos 330^{\circ} = \frac{\sqrt{3}}{2}$ (330° in QIV, reference angle 30°)
h) $\sin 270^{\circ} = -1$
i) $\tan 225^{\circ} = \tan 45^{\circ} = 1$ (225° in QIII, reference angle 45°)
j) $\cos 360^{\circ} = 1$
k) $\sin 240^{\circ} = -\frac{\sqrt{3}}{2}$ (240° in QIII, reference angle 60°)

I) $\tan 300^\circ = -\sqrt{3}$ (300° in QIV, reference angle 60°)