

## Chapter 4: Ratios and rates Solution

### 4A Introducing ratios

1.

(a) mg : g

$$1 \text{ mg} : 1 \text{ g} = 1 : 1000$$

(b) km : m

$$1 \text{ km} : 1 \text{ m} = 1 : 1000$$

(c) second : minute

$$1 \text{ s} : 1 \text{ min} = 1 : 60$$

(d) kilogram : tonne

$$1 \text{ kg} : 1 \text{ tonne} = 1 : 1000$$

(e) litre : millilitre

$$1 \text{ L} : 1 \text{ mL} = 1 : 1000$$

(f) day : week

$$1 \text{ day} : 1 \text{ week} = 1 : 7$$

2.

(a) 100 g : 1 kg

- 1 kg = 1000 g.
- So 100 g : 1000 g = 100 : 1000 = 1 : 10.

(b) 500 ml : 2 L

- 2 L = 2000 ml.
- Ratio: 500 : 2000 = 1 : 4.

(c) 120 cm : 1.5 m

- 1.5 m = 150 cm.
- Ratio: 120 : 150 = 12 : 15 = 4 : 5.

(d) 800 g : 4 kg

- 4 kg = 4000 g.
- Ratio: 800 : 4000 = 1 : 5.

(e) 30 mm : 2 m

- 2 m = 2000 mm.
- Ratio: 30 : 2000 = 3 : 200.

(f) 5 g : 100 g

- Ratio: 5 : 100 = 1 : 20.

3.

a) 25, b) 21, c) 5, d) 18, e) 24, f) 3.75 (or  $15/4$ ), g) 6, h)  $28/3$  ( $\approx 9.33$ ), i)  $136/7$  ( $\approx 19.43$ ).

4.

- Protein : Fibre = 3 : 7
- If protein = 2100 kg, then fibre = 4900 kg.

5.

Ratio (gold : other) = 14 : 10

(a) If 5 g gold  $\rightarrow$  other =  $25/7$  g  $\approx 3.57$  g

(b) 12 ct gold ratio = 12 : 12 = 1 : 1.

6.

Original ratio (gold : silver : platinum) = 18 : 3 : 3  $\rightarrow$  simplest = 6 : 1 : 1

(a) 6:1:1

(b) If 4 g gold  $\rightarrow$  silver =  $2/3$  g

(c) If 2 g gold  $\rightarrow$  platinum =  $1/3$  g.

7.

**Fertilizer (N : P : K = 10 : 5 : 3)**

(a) If 20 kg N  $\rightarrow$  P=10 kg, K=6 kg

(b) If 30 kg K  $\rightarrow$  N=100 kg, P=50 kg

(c) Doubling N  $\rightarrow$  20:5:3 (cannot simplify further).

8.

**Cleaning Solution (Water : Vinegar = 4 : 1)**

(a) 600 mL water  $\rightarrow$  150 mL vinegar

(b) For 1 L total  $\rightarrow$  water = 800 mL, vinegar = 200 mL.

## 4B Simplifying ratios

1.

(a)  $45 : 60 = 3 : 4$

(b)  $150 : 250 = 3 : 5$

(c)  $36 : 54 = 2 : 3$

(d)  $18 : 42 = 3 : 7$

(e)  $90 : 135 = 2 : 3$

(f)  $0.4 : 0.6 = 2 : 3$

(g)  $0.18 : 0.54 = 1 : 3$

(h)  $0.015 : 0.045 = 1 : 3$

(i)  $0.2 : 0.08 = 5 : 2$

(j)  $0.75 : 3 = 1 : 4$

(k)  $2 : 1.4 = 10 : 7$

(l)  $1.2 : 4 = 3 : 10$

(m)  $8 : 12 : 20 = 2 : 3 : 5$

(n)  $6 : 3.6 : 1.2 = 5 : 3 : 1$

(o)  $36 : 72 : 108 = 1 : 2 : 3$

(p)  $400 : 500 : 1000 = 4 : 5 : 10$

(q)  $7 : 21 : 14 = 1 : 3 : 2$

(r)  $4.5 : 2.7 : 0.9 = 5 : 3 : 1$

(s)  $54 : 81 : 108 = 2 : 3 : 4$

(t)  $240 : 360 : 720 = 2 : 3 : 6$ .

2.

- (a)  $\frac{4}{5} : \frac{7}{8} = 32 : 35$ .
- (b)  $\frac{84}{39} : \frac{1}{2} = 56 : 13$ .
- (c)  $\frac{16}{32} : \frac{75}{4} = 2 : 75$ .
- (d)  $\frac{74}{13} : \frac{98}{6} = 222 : 637$ .
- (e)  $\frac{12}{6} : \frac{63}{25} = 50 : 63$ .
- (f)  $\frac{34}{78} : \frac{4}{3} = 17 : 52$ .
- (g)  $\frac{9}{5} : \frac{89}{12} = 108 : 445$ .
- (h)  $\frac{54}{9} : \frac{6}{3} = 3 : 1$ .
- (i)  $3\frac{4}{6} : \frac{54}{9} = 11 : 18$ .
- (j)  $\frac{32}{98} : 6\frac{6}{66} = 176 : 3283$ .
- (k)  $\frac{53}{26} : \frac{94}{17} = 901 : 2444$ .
- (l)  $\frac{84}{32} : \frac{9}{7} = 49 : 24$ .

3.

- (a)  $42xyz : 21yz = 2x : 1$ .
- (b)  $15a : 50c = 3a : 10c$ .
- (c)  $50x : x^2 = 50 : x$ .
- (d)  $88a : 24b = 11a : 3b$ .
- (e)  $6g : xg = 6 : x$ .
- (f)  $45x^2t : 135xt^2 = x : 3t$ .
- (g)  $76x : 129y = 76x : 129y$  (already simplest).
- (h)  $xyz : x^3y^4 = z : x^2y^3$ .

#### 4C Dividing a quantity in a given ratio

1.

1a. 250 in 2:3 → 100 and 150

1b. 1200 in 5:7 → 500 and 700

1c. 80 in 4:6 → 32 and 48

1d. 90 in 2:3 → 36 and 54

1e. 500 in 7:8 →  $\approx 233.33$  and  $\approx 266.67$

1f. 65 in 9:11 → 29.25 and 35.75

1g. 300 in 2:3:5 → 60, 90, 150

1h. 600 in 1:4:5 → 60, 240, 300

1i. 24 in 2:3:4 →  $5\frac{1}{3}$ , 8,  $10\frac{2}{3}$

2.

Total mixture = 1.4 L

Paint needed: **0.4 L**

3.

Sides: **22.5 cm, 30 cm, 37.5 cm**

4.

$$\text{Ratio} = \frac{\pi r^2 h}{2\pi r^2 + 2\pi r h} \implies r h : 2(r^2 + r h).$$

5.

$$\text{Area : Perimeter} = s^2 : 4s = s : 4.$$

6.

Shaded area = **42 cm<sup>2</sup>**

7.

**23.2 cm, 18.6 cm, 13.9 cm**

8.

Apple slices = **9**

9.

Areas: **16 m<sup>2</sup>, 24 m<sup>2</sup>, 32 m<sup>2</sup>**

10.

Teachers = **30**

11.

Ratio = **5 : 6**

12.

Flour = **1.5 kg**

Sugar = **1.0 kg**

13.

Distances: **80, 100, 120** miles

14.

a) **\$12,000 Bonus in 3:4:5**

**\$3,000, \$4,000, \$5,000**

**14b) Triangular Garden 7:8:9, Perimeter 96 m**

**Sides: 28 m, 32 m, 36 m**

15.

Acid = **1.5 L**

Water = **3.5 L**

## 4D Scale drawings

1.

### Blueprint (Scale 1:100)

- (a) 2 cm  $\rightarrow$  2 m
- (b) 8 mm  $\rightarrow$  0.8 m
- (c) 4.5 cm  $\rightarrow$  4.5 m
- (d) 10 cm  $\rightarrow$  10 m

2.

### Map (Scale 1:10,000 = 1 cm $\rightarrow$ 100 m)

- (a) 100 m  $\rightarrow$  1 cm
- (b) 20 km  $\rightarrow$  2 m
- (c) 200 km  $\rightarrow$  20 m
- (d) 85 km  $\rightarrow$  8.5 m

3.

### Car Model (Scale 1:50)

- (a) 3 cm  $\rightarrow$  1.5 m
- (b) 10.5 cm  $\rightarrow$  5.25 m
- (c) 7 cm  $\rightarrow$  3.5 m
- (d) 20 cm  $\rightarrow$  10 m

4.

### Same-Unit Ratios

- (a) 3 cm : 300 m = 1 : 10,000
- (b) 10 mm : 1 m = 1 : 100
- (c) 15 mm : 450 cm = 1 : 300

5.

### Same-Unit Ratios

- (a) 8 cm : 800 mm = 1 : 10
- (b) 20 mm : 2 m = 1 : 100
- (c) 18 mm : 540 cm = 1 : 300

6.

### Rectangular Park (100 m $\times$ 60 m)

- (a) Scale 1:500  $\rightarrow$  20 cm  $\times$  12 cm
- (b) 5 mm to 1 m  $\rightarrow$  50 cm  $\times$  30 cm

7.

### Swimming Pool (25 m $\times$ 10 m)

- (a) Scale 1:250  $\rightarrow$  10 cm  $\times$  4 cm
- (b) 2 mm to 1 m  $\rightarrow$  5 cm  $\times$  2 cm

#### 4E Introducing rates

1.  
(a) 5 apples per \$; (b) 25 mi/gal; (c) 0.25 km/min
2.  
(a) 2 kg/\$; (b) 6.25 km/L; (c) 0.2 mi/min
3.  
8 minutes
4.  
30 km/h
5.  
(a) ~20.8 km/h, (b) ~28.8 minutes for 10 km
6.  
75 km/h
7.  
30 minutes
8.  
(a) 5 hr, (b) 2.5 hr
9.  
Bob must leave  $\frac{D}{90}$  hr after 7 a.m. (depends on distance D)
10.  
~6:38 a.m. start for Sarah
11.  
~9:09 a.m. start for Mike
12.  
5 hr total
13.  
(a) 1 hr 30 min, (b) 3 hr 45 min
14.  
~9.09 m/s
15.  
600 units/hr
16. 4 km/h



## 4F unitary method

1.
  - 1(a) \$6.00
  - 1(b) 75 min
  - 1(c) 35 s/week
  - 1(d) 250 kg
  - 1(e) 15 km
  
2.
  - 2(a) 25 c/min
  - 2(b)  $\sim 2.22$  c/s
  - 2(c) 600 L/h
  - 2(d) 30 L/h
  - 2(e) 24 kg/year
  - 2(f) 2.1 kg/week
  - 2(g) 9 kg/\$
  - 2(h) 7.5 c/mm
  
3.

500 g tub is cheaper per gram
  
4.

(a) 630 km, (b) 16.67 L
  
5.

(a)  $\sim 19.7$  in, (b)  $\sim 19.69$  in
  
6.

(a) 2.5 h, (b) 2520 km
  
7.

(a) 40 min, (b) 10800 bottles
  
8.

$\sim 19.46$  km/h
  
9.

96 km/h
  
10.

30 km/h
  
11.

5 km/h
  
12.

$\sim 9.80$  m/s

13.  
72 km/h
14.  
148.16 km
15.  
70 mph
16.  
625 units/h
17.  
~6.67 items/min
18.  
~80.45 km/h
19.  
~1.11 m/s
20.  
3 hours