Chapter 4: Linear relations

4A Introduction to linear equations

1.

m=5 c=-7	m=-1/3 c=2	m=-1 c=0	m=2 c=3
m=2 c=4	m=2/3 c=-5/3	m=4/5 c=$\sqrt{3}$	m=8 c=-12
m=-3/2 c=3	m=-21/2 c=15/2	m=4 c=-3	m=7/5 c=-2

2.

True	True	False	False

3.

False	False	False	False

4.

x value	y value
-3	-7
-2	-5
-1	-3
0	-1
1	1
2	3
3	5

5.

x value	y value
-3	-1.5
-2	-2
-1	-2.5
0	-3
1	-3.5
2	-4
3	-4.5

6.

y=2x-1 is a straight line with a positive slope of 2 and y=-1/2x -3 is a straight line with a negative slope of -1/2, the two line are perpendicular to each other.

7.

x value	y value
-3	-10
-2	-6
-1	-2
0	2
1	6
2	10
3	14

8.

x value	y value
-3	-4
-2	0
-1	4
0	8
1	12
2	16
3	20

9.

The two lines are parallel because they have the same slope but different y-intercepts.

4B Graphing straight lines using intercepts

1. Sketch the graphs and show the x and y-intercepts for the following equations:

x-intercept = 1/3 y-intercept = -1	x-intercept = 32 y-intercept = 8	x-intercept = -2 y-intercept = 10
x-intercept = 2 y-intercept = 4	x-intercept = 9/4 y-intercept = 9	x-intercept = 3 y-intercept = 3
x-intercept = 3 y-intercept = -15	x-intercept = 9 y-intercept = 3	x-intercept = 2 y-intercept = -6
x-intercept = -6/5 y-intercept = 2	x-intercept = -8 y-intercept = 4	x-intercept = -3 y-intercept = 1
x-intercept = 2.5 y-intercept = 1	x-intercept = 2 y-intercept = 3	x-intercept = 3 y-intercept = -7

2.

a. The initial height of the object is 100 meters.

b. The height becomes 0 meters after 20 seconds

3.

y = 2x +10	$y = \frac{1}{3}x - 3$	$y = c - \frac{x}{3}$
y = 12-2x	y = b + x	$y = 8 - \frac{1}{2}x$

4.

The fuel starts at 50 liters and decreases linearly, reaching 0 liters in 5 days.

The equation is y=-10x + 50 showing a decreasing linear trend.

5.

The water starts at 10 liters and increases at a rate of 5 liters per hour.

The equation is y = 5x + 10, showing an increasing linear trend.

6.

a. Y-intercept is 40

   X-intercept is $-\frac{8}{3}$

c. The y-intercept (40) represents the initial cost of the gym membership, which is the joining fee before the any monthly payments are made. This means that even if a person does not stay for any months, they still need to pay $40 to join.

4C Lines with one intercept

1. n/a

2. n/a

3.

y=3x	$y=\frac{2}{3}x$	y=-2.5x
$y=\frac{1}{2}x$	y=-2x	y=2x

4.

m = 5.11

5.

vertical line passing through (3,4): x=3

Horizontal line passing through (3,4): y=4

Line passing through the origin and (3,4): $y=\frac{4}{3}x$

6.

The line passes through (0,0) and (5,-3). The equation is $y=-\frac{3}{5}x$

4D Gradient

1.

$m=-\frac{3}{4}$	$m=\frac{3}{5}$	$m=-\frac{4}{3}$
$m=\frac{5}{7}$	m=1	$m=-\frac{1}{2}$

2.

$m=\frac{4}{3}$	$m=\frac{1}{2}$	m=2
$m=\frac{1}{2}$	$m=\frac{4}{5}$	m=1
$m=\frac{3}{2}$	m=1	m=-3
m=3	m=-1	m=2.5

3.

m=2

4.

m=-2

5.

m=-2

6.

m=2

7.

$\frac{1600}{3}$

8.

a. (3,10)

b. (-3,15)

c. (6,6)

d. (1,7)

e. (2,6)

9. the line with gradient $\frac{6}{5}$ is steeper

10.

a. x = 4 then y = 7

x = 5 then y = 9.5

b. x-intercept is x = 1.2

4F Gradient intercept form

1.

m=5 b=7	m=-2 b=1	m=3 b=-4	m=3 b=-6
not relevant	m=$-\frac{1}{2}$ b=-5	m=-2 b=5	m=1.5 b=3
m=-3 b=6	m=0.5 b=-2	m=-2 b=7	m=-2 b=$\frac{13}{4}$

2.

a. y=$-\frac{x}{2}+\frac{5}{2}$
b. y=3x-4
c. y=3x+4
d. 2x-y=3
e. 3x-5y=10 or -3x+5y=-10
f. y=4x-7
g. $\frac{1}{2}$x-y=4
h. y=-2x+3

3.

gradient: -2 y-intercept : 3
gradient: $-\frac{1}{2}$ y-intercept : -2

gradient: $-\frac{1}{3}$ y-intercept : 0
gradient: $-\frac{1}{4}$ y-intercept : 2

4G Find the equation of a line

1.

y = 4x + 3

2.

y = $-\frac{3}{4}x + 11$

3.

y = 7x

4.

y = -2x + 8

5.

y = $\frac{2}{3}x + \frac{22}{3}$

6.

$y=\frac{4}{3}x+\frac{2}{3}$	$y=-\frac{9}{5}x-\frac{2}{5}$	y=2x+1
$y=\frac{1}{2}x-3$	y=x+7	$y=-\frac{1}{2}x-\frac{1}{2}$
y=x+6	$y=\frac{2}{3}x-\frac{14}{3}$	y=2x
$y=\frac{1}{2}x-2$	y=3x-13	y=-3x+4

7.

the rule for the speed of the car is : y=-15x+95

the initial speed of the car is 95km/h

8.

a. c=3

b. c=3

c. no, it does not matter which point is used.

4H Midpoint and length of a segment

1.

(-1,2)	(2,10)	(4,1)
(0,2)	(4,2)	(0,1)
(2,-4)	(2.5,5)	(2,3.5)

2.

a. (5a,3b)

b. $(\frac{2x+1.5y-3}{2}, y+2)$

c. $(\frac{2x+3.4y-2}{2}, y+3)$

3.

(3,4)	(-1,-1)
(10,5)	(-2,-3)
(4,8)	(-7,-5)

4.

5.00	7.81	9.90
8.06	8.06	7.62

5.

B is further from A

6.

Midpoint P is (6,1)

Distance from B to p : 5.7

7.

perimeter of the triangle : 24.2

8.

trapezium perimeter : 16.4

9.

a. (-6,4)

b. (-1,2)

c. (2,10)

4I Perpendicular and parallel lines

1.

these lines are parallel	these lines are parallel

2.

$y = 3x + 5$

3.

$y = -5x - 2$

4.

these lines are perpendicular	these lines are not perpendicular

5.

$y = -\frac{1}{4}x + \frac{11}{2}$

6.

$y = -2$

7.

$x = 3$

8.

$y = -2x + 11$

9.

yes, they are perpendicular.

10.

y = $\frac{1}{3}x$

11.

No, they are not.

4J Linear modelling

1.

a. intercept on vertical axis: 5

b. gradient : $\frac{35}{6}$ or 5.83

c. equation of the line: y = $\frac{35}{6}x + 5$

d. money after 3.5 hours : $30.42

e. time to earn $19 : 2.4 hours

2.

a.

500	460	420	380	340

b.

y = -4x + 500

c.

the tank will be empty after 125 minutes

d.

after 25 minutes, 400 liters of water will remain

3.

b. V = -40t + 1200

c. water is being used at 40L/min

d. 800 liters

e. 15 minutes

4.

a. $50 per day

b. C = 60t + 100

4K Graphical solution to simultaneous equations

1.

1 solution	no solution	1 solution
no solution	1 solution	no solution
1 solution	1 solution	1 solution

2.

No solution, because the lines are parallel.

3.

(-5,-6)	$(\frac{7}{3}, \frac{2}{3})$
$(\frac{4}{3}, \frac{11}{3})$	$(\frac{9}{7}, -\frac{13}{7})$

4.

a. NO

b. NO

C. No

d. NO

5.

b. 125 cupcakes

6. 6.67 hours or 6 hours 40 minutes