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Homework Answer

7A Exploring parabolas

1.

Equation	Value of a	Wider / Narrower
$y = 2x^2$	2	Narrower
$y = 1.5x^2$	1.5	Narrower
$y = 0.7x^2$	0.7	Wider
$y = 0.02x^2$	0.02	Wider
$y = \frac{3}{2}x^2$	$\frac{3}{2} = 1.5$	Narrower
$y = \frac{12}{7}x^2$	$\frac{12}{7} \approx 1.71$	Narrower
$y = \frac{x^2}{3}$	$\frac{1}{3}$	Wider
$y = \frac{2}{5}x^2$	$\frac{2}{5} = 0.4$	Wider

2.

Equation	a	Wider / Narrower	Turning point
$y = 5x^2$	5	Narrower	Minimum
$y = -5x^2$	-5	Narrower	Maximum
$y = \frac{1}{3}x^2$	$\frac{1}{3}$	Wider	Minimum
$y = -\frac{2}{5}x^2$	$-\frac{2}{5}$	Wider	Maximum
$y = -1.2x^2$	-1.2	Narrower	Maximum
$y = -\frac{3}{4}x^2$	$-\frac{3}{4}$	Wider	Maximum
$y = 1.6x^2$	1.6	Narrower	Minimum
$y = \sqrt{5}\,x^2$	$\sqrt{5} \approx 2.24$	Narrower	Minimum

3.

i. (0, 2); minimum ii. X=0 iii. (-1, 0) (1, 0) iv. (0, -2)	i. (1.5, -2.25); minimum ii. X=1.5 iii. (0, 0) (3, 0) iv. (0, 0)
i. (1, 3); maximum ii. X=1 iii. (0, 0) (2, 0) iv. (0, 0)	i. (0.625, 1.125); maximum ii. X=0.625 iii. (0.25, 0) (1, 0) iv. (0, -2)

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7B Sketching parabolas using transformations

1.

Equation	Shift	Turning point
$y = x^2 + 4$	4↑	(0, 4)
$y = x^2 - 2.5$	2.5↓	(0, –2.5)
$y = x^2 + \frac{1}{3}$	$\frac{1}{3}$↑	$(0, \frac{1}{3})$
$y = x^2 - \frac{7}{8}$	$\frac{7}{8}$↓	$(0, -\frac{7}{8})$
$y = x^2 + 0.4$	0.4↑	(0, 0.4)
$y = x^2 - 3$	3↓	(0, –3)
$y = x^2 + \sqrt{2}$	$\sqrt{2}$↑	$(0, \sqrt{2})$
$y = x^2 - \frac{5}{4}$	1.25↓	(0, –1.25)
$y = x^2 - 4.6$	4.6↓	(0, –4.6)
$y = x^2 - \sqrt{6}$	$\sqrt{6}$↓	$(0, -\sqrt{6})$
$y = x^2 - \frac{1}{9}$	$\frac{1}{9}$↓	$(0, -\frac{1}{9})$
$y = x^2 + 3.5$	3.5↑	(0, 3.5)
$y = 4 + x^2$	4↑	(0, 4)
$y = 5 + x^2$	5↑	(0, 5)
$y = -3 + x^2$	3↓	(0, –3)
$y = -1.5 + x^2$	1.5↓	(0, –1.5)

2.

Equation	Shift	Turning point
$y = (x - 3)^2$	3 →	(3, 0)
$y = (x + 2)^2$	2 ←	(–2, 0)
$y = (x - 1.2)^2$	1.2 →	(1.2, 0)
$y = (x + 6)^2$	6 ←	(–6, 0)
$y = (x - \frac{17}{8})^2$	2.125 →	$(\frac{17}{8}, 0)$
$y = (x - 14)^2$	14 →	(14, 0)
$y = (x - 4.4)^2$	4.4 →	(4.4, 0)
$y = (x + \frac{13}{5})^2$	2.6 ←	(–2.6, 0)
$y = (x - 1.1)^2$	1.1 →	(1.1, 0)
$y = (x + 4.05)^2$	4.05 ←	(–4.05, 0)
$y = (x - \frac{3}{14})^2$	0.214 →	$(\frac{3}{14}, 0)$
$y = (x + \frac{17}{12})^2$	1.417 ←	(–1.417, 0)
$y = (x + 3.5)^2$	3.5 ←	(–3.5, 0)
$y = (x - 0.5)^{2*}$	0.5 →	(0.5, 0)
$y = (x + 5.7)^2$	5.7 ←	(–5.7, 0)
$y = (x - 0.5)^{2*}$	0.5 →	(0.5, 0)

* (The original line read "$(-½ + x)^2$"; the interpretation here is $(x - 0.5)^2$.)

3.

Equation	Shift (→/←, ↑/↓)	Turning point
$y = (x-4)^2 + 3$	4→, 3↑	(4, 3)
$y = (x+1)^2 - 5$	1←, 5↓	(–1, –5)
$y = x^2 + 6$	0 , 6↑	(0, 6)
$y = (x-3)^2 + 1$	3→, 1↑	(3, 1)
$y = (x-3)^2 - 9$	3→, 9↓	(3, –9)
$y = 2(x+1)^2 - 6$	1←, 6↓	(–1, –6)
$y = -(x+5)^2 - 1$	5←, 1↓	(–5, –1)
$y = (x+1)^2 - 2$	1←, 2↓	(–1, –2)
$y = (x-4)^2 + 6$	4→, 6↑	(4, 6)
$y = (x+3)^2 - 7$	3←, 7↓	(–3, –7)
$y = 3(x-1)^2 + 5$	1→, 5↑	(1, 5)
$y = (x-1)^2 + 6$	1→, 6↑	(1, 6)
$y = -(x+2)^2 - \frac{1}{2}$	2←, 0.5↓	(–2, –0.5)
$y = -(x-3)^2 + 5$	3→, 5↑	(3, 5)
$y = (x+4)^2 + 3$	4←, 3↑	(–4, 3)
$y = (x-1)^2 - 1$	1→, 1↓	(1, –1)

4.

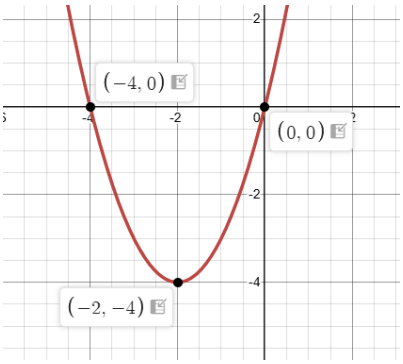
Equation	Turning point	Max / Min
$y = -(x-3)^2 + 4$	(3, 4)	maximum
$y = (x+6)^2 - 7$	(–6, –7)	minimum
$y = \frac{3}{2}x^2$	(0, 0)	minimum
$y = (x+4)^2$	(–4, 0)	minimum
$y = 11 - (x-4)^2$	(4, 11)	maximum
$y = 7 + (x-5)^2$	(5, 7)	minimum
$y = (x+5)^2 - 3$	(–5, –3)	minimum
$y = -2.5x^2$	(0, 0)	maximum
$y = (x+9)^2 - 1$	(–9, –1)	minimum
$y = -\frac{3}{8}(x-5)^2$	(5, 0)	maximum
$y = -(x+2)^2$	(–2, 0)	maximum
$y = 2.5x^2$	(0, 0)	minimum
$y = -3(x+2)^2 + 9$	(–2, 9)	maximum
$y = (x+4)^2 - 8$	(–4, –8)	minimum
$y = 10 - (x-3)^2$	(3, 10)	maximum
$y = -6x^2$	(0, 0)	maximum

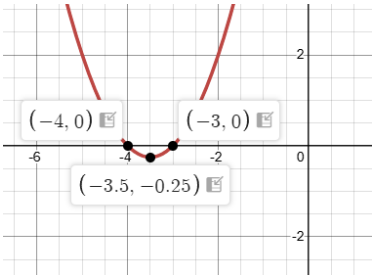
5.

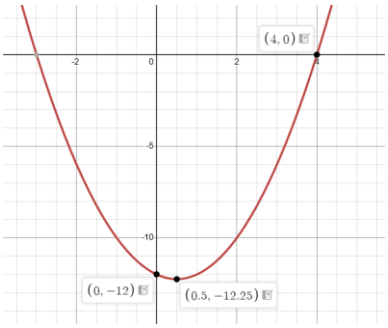
Prompt	New rule
a) 6 →	$y = (x-6)^2$
b) 4 ↓	$y = x^2 - 4$
c) 3 ↑	$y = x^2 + 3$
d) 8 ←, 2 ↑	$y = (x+8)^2 + 2$
e) 2 →, 7 ↓	$y = (x-2)^2 - 7$
f) 4 ←, 6 ↓	$y = (x+4)^2 - 6$
g) reflect x-axis, 1 ←	$y = -(x+1)^2$
h) 3 ←, 4 ↓	$y = (x+3)^2 - 4$
i) reflect x-axis, 4 ↑	$y = -x^2 + 4$
j) 2 →, 7 ↑	$y = (x-2)^2 + 7$
k) 7 ←, 1 ↓	$y = (x+7)^2 - 1$
l) reflect x-axis	$y = -x^2$
m) 8 ↑	$y = x^2 + 8$
n) 9 →, 5 ↓	$y = (x-9)^2 - 5$
o) reflect x-axis, 7 →	$y = -(x-7)^2$
p) 2 ←	$y = (x+2)^2$
q) reflect x-axis, 4 ↑	$y = -x^2 + 4$
r) 1 ←, 8 ↓	$y = (x+1)^2 - 8$
s) 10 →	$y = (x-10)^2$
t) 6 →	$y = (x-6)^2$

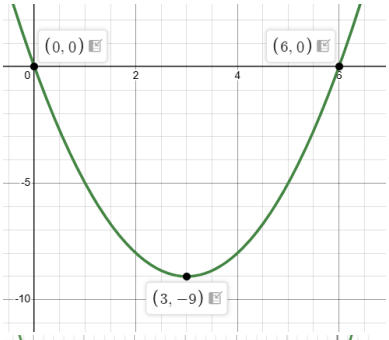
7C Sketching parabolas using factorisation

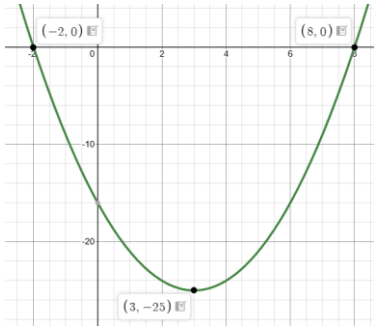
1.

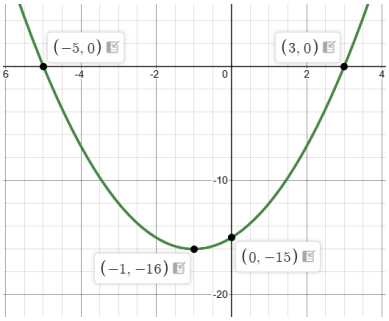


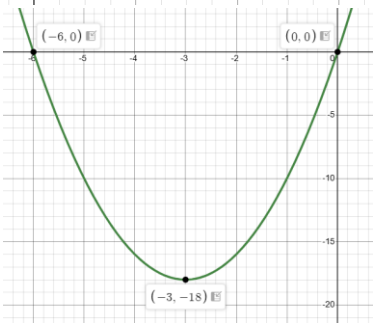


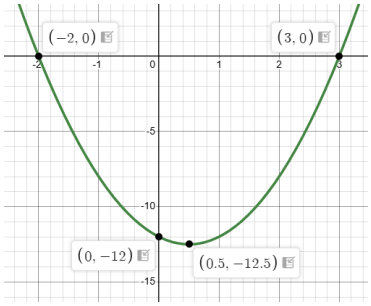


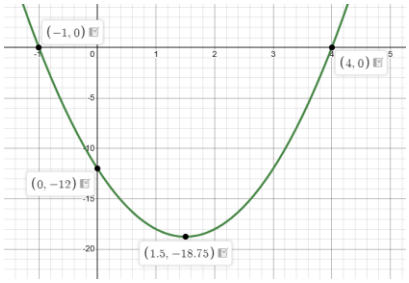


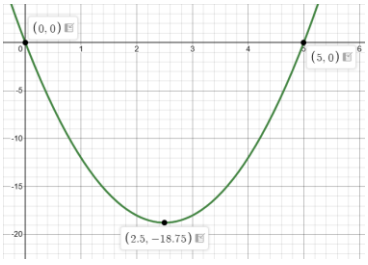


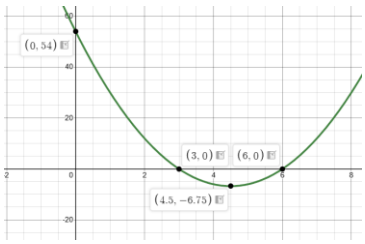


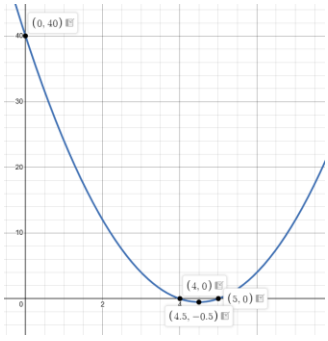












2.

Quadratic	Vertex form	Turning point
$y = x^2 + 6x - 2$	$(x+3)^2 - 11$	$(-3, -11)$
$y = x^2 + 4x + 1$	$(x+2)^2 - 3$	$(-2, -3)$
$y = x^2 + 8x - 3$	$(x+4)^2 - 19$	$(-4, -19)$
$y = x^2 - 8x - 2$	$(x-4)^2 - 18$	$(4, -18)$
$y = x^2 - 10x + 2$	$(x-5)^2 - 23$	$(5, -23)$
$y = x^2 - 4x + 2$	$(x-2)^2 - 2$	$(2, -2)$
$y = x^2 + 12x - 5$	$(x+6)^2 - 41$	$(-6, -41)$
$y = x^2 - 6x + 10$	$(x-3)^2 + 1$	$(3, 1)$
$y = x^2 + 4x + 9$	$(x+2)^2 + 5$	$(-2, 5)$
$y = 2x^2 + 12x + 4$	$2(x+3)^2 - 14$	$(-3, -14)$
$y = 2x^2 - 6x - 2$	$2\left(x - \frac{3}{2}\right)^2 - \frac{13}{2}$	$\left(\frac{3}{2}, -\frac{13}{2}\right)$
$y = 2x^2 - 8x + 6$	$2(x-2)^2 - 2$	$(2, -2)$
$y = 2x^2 - 4x - 10$	$2(x-1)^2 - 12$	$(1, -12)$
$y = 2x^2 + 4x + 6$	$2(x+1)^2 + 4$	$(-1, 4)$
$y = 2x^2 + 6x - 10$	$2\left(x + \frac{3}{2}\right)^2 - \frac{29}{2}$	$\left(-\frac{3}{2}, -\frac{29}{2}\right)$

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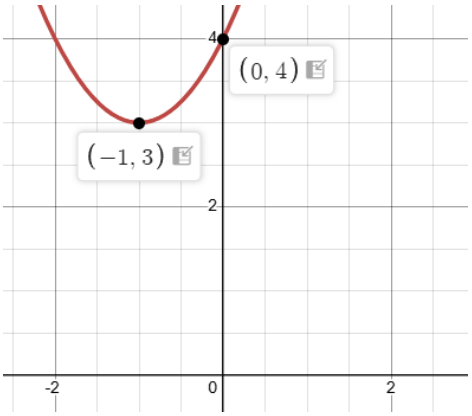
7D Sketching parabolas by completing the square

1.

Equation	x-intercept(s) (set $y = 0$)	y-intercept (set $x = 0$)
$y = (x+2)^2 - 4$	$(0,0),\ (-4,0)$	$(0,0)$
$y = (x-3)^2 + 4$	none (no real roots)	$(0,13)$
$y = (x-2)^2 + 6$	none	$(0,10)$
$y = (x-6)^2 - 9$	$(3,0),\ (9,0)$	$(0,27)$
$y = (x+1)^2 + 7$	none	$(0,8)$
$y = (x-5)^2 - 2$	$(5-\sqrt{2},0),\ (5+\sqrt{2},0)$	$(0,23)$
$y = (x+2)^2 - 3$	$(-2-\sqrt{3},0),\ (-2+\sqrt{3},0)$	$(0,1)$
$y = (x-8)^2 + 2$	none	$(0,66)$
$y = (x+2)^2 + 12$	none	$(0,16)$
$y = (x-5)^2 + 6$	none	$(0,31)$
$y = (x+7)^2 + 1$	none	$(0,50)$
$y = (x-2)^2 - 4$	$(0,0),\ (4,0)$	$(0,0)$
$y = (x+4)^2 + 8$	none	$(0,24)$
$y = (x-5)^2 + 6^*$	none	$(0,31)$
$y = (x+3)^2 - 9$	$(-6,0),\ (0,0)$	$(0,0)$

2.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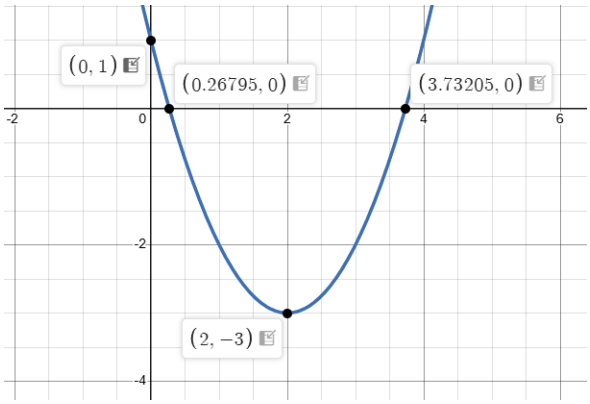
(−1, 3)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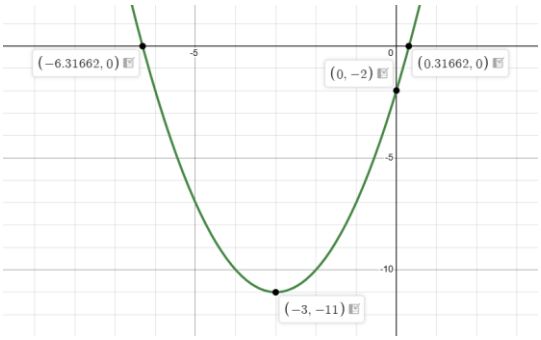
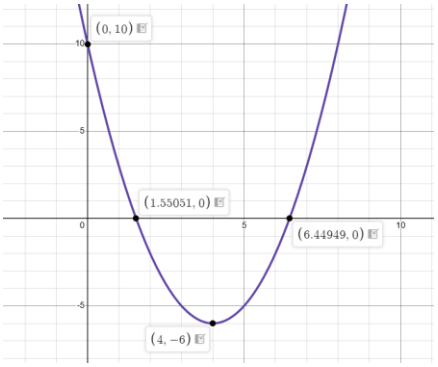
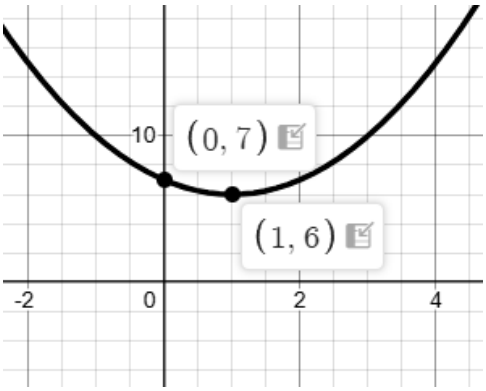
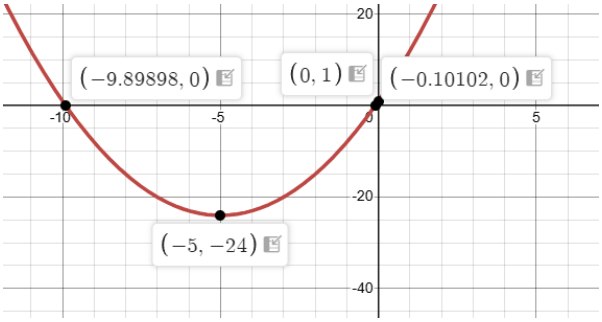
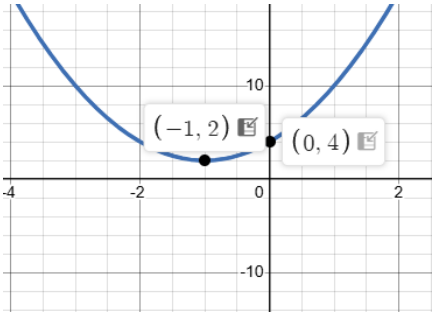
(3.73205, 0)
(2, −3)

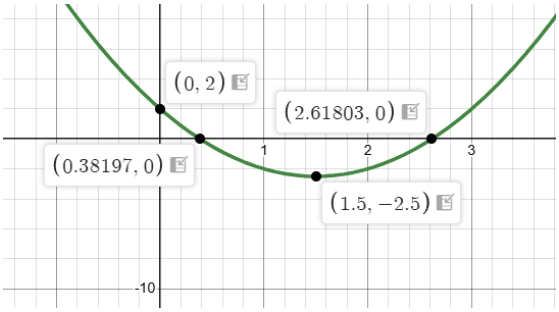

(−6.31662, 0)
(0, −2)
(0.31662, 0)
(−3, −11)


(0, 10)
(1.55051, 0)
(6.44949, 0)
(4, −6)


(0, 7)
(1, 6)


(−9.89898, 0)
(0, 1)
(−0.10102, 0)
(−5, −24)


(−1, 2)
(0, 4)


(0, 2)
(2.61803, 0)
(0.38197, 0)
(1.5, −2.5)

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7E Sketching parabolas using the quadratic formula

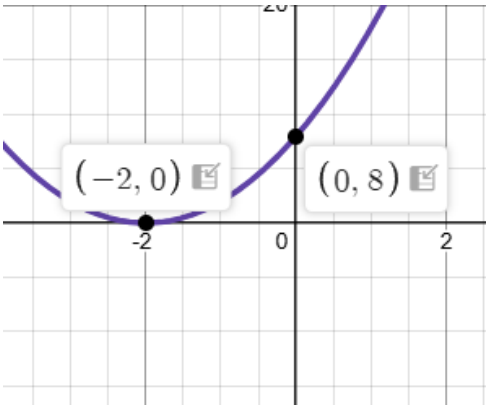
1.

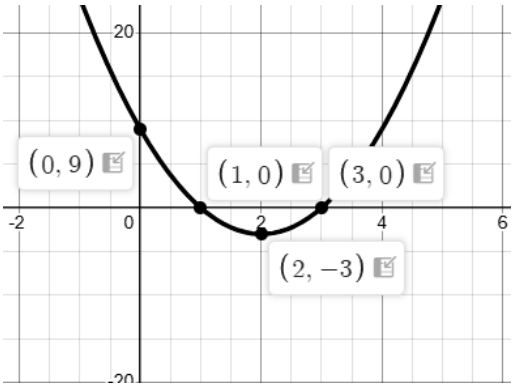
Quadratic	a	b	c	D	No. of real x-intercepts
x^2+6x+8	1	6	8	$36-32=4>0$	2
x^2-5x+9	1	–5	9	$25-36=-11<0$	0
$-x^2+2x+7$	–1	2	7	$4+28=32>0$	2
$3x^2-12x+12$	3	–12	12	$144-144=0$	1
$4x^2+x+1$	4	1	1	$1-16=-15<0$	0
$2x^2-14x+24$	2	–14	24	$196-192=4>0$	2
$x^2-7x+13$	1	–7	13	$49-52=-3<0$	0
$-2x^2+5x+3$	–2	5	3	$25+24=49>0$	2
$5x^2-20x+20$	5	–20	20	$400-400=0$	1

2.

Quadratic	a	b	x-coordinate	y-coordinate	Turning point (h,k)
$3x^2+4x+10$	3	4	$-\frac{4}{3}=-\frac{2}{3}$	$y=\frac{26}{3}$	$(-\frac{2}{3},\frac{26}{3})$
$6x^2-18x+12$	6	–18	$\frac{18}{12}=\frac{3}{2}$	$y=-\frac{3}{2}$	$(\frac{3}{2},-\frac{3}{2})$
$2x^2+3x-2$	2	3	$-\frac{3}{4}$	$y=-\frac{25}{8}$	$(-\frac{3}{4},-\frac{25}{8})$
$x^2-10x+25$	1	–10	5	0	$(5,0)$
x^2+3x+5	1	3	$-\frac{3}{2}$	$y=\frac{11}{4}$	$(-\frac{3}{2},\frac{11}{4})$
$-5x^2+15x-10$	–5	15	$\frac{15}{-10}=\frac{3}{2}$	$y=\frac{5}{4}$	$(\frac{3}{2},\frac{5}{4})$

3.


(−2, 0)
(0, 8)
-2
0
2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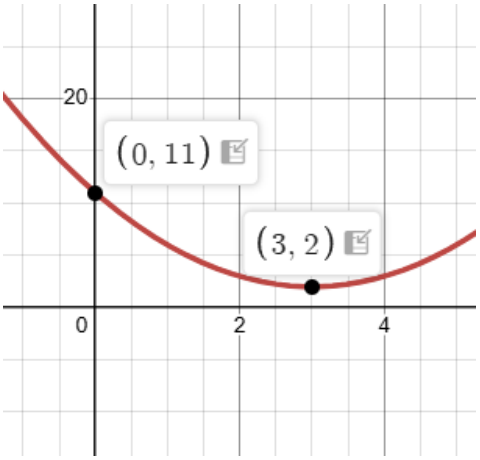
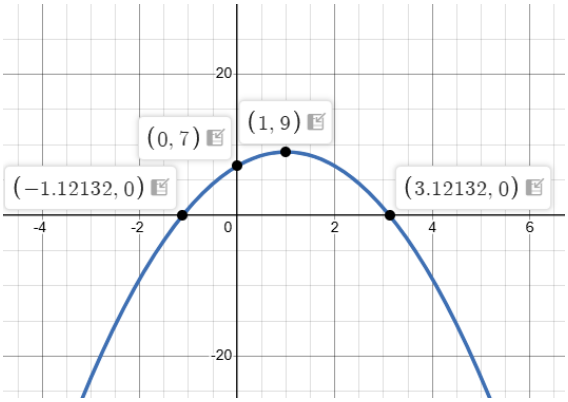
(0, 9)
(1, 0)
(3, 0)
(2, −3)
-2
0
2
4
6
-20


20
(0, 11)
(3, 2)
0
2
4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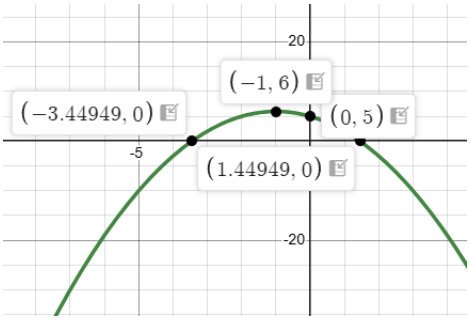
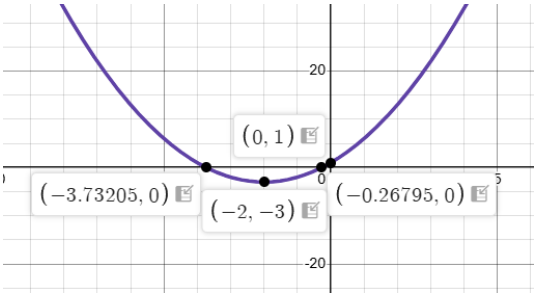
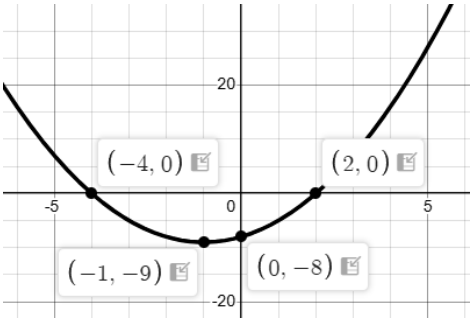
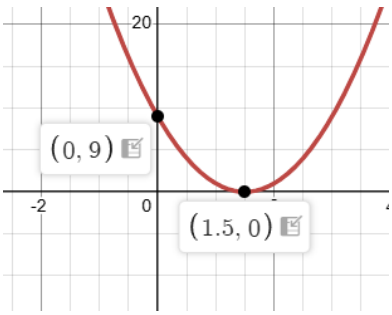
(1, 9)
(−1.12132, 0)
(3.12132, 0)
-4
-2
0
2
4
6
-20


20
(−1, 6)
(−3.44949, 0)
(0, 5)
(1.44949, 0)
-5
-20

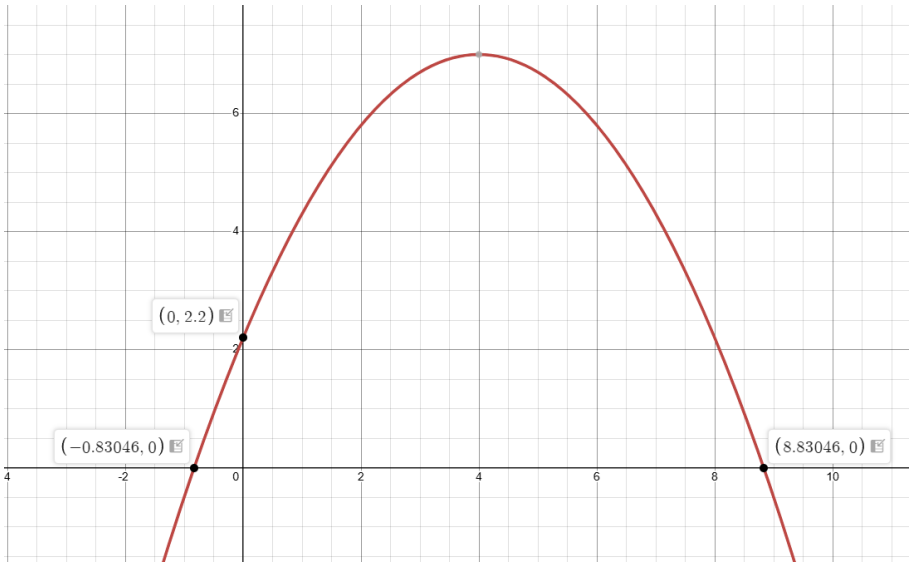

20
(0, 1)
(−3.73205, 0)
(−2, −3)
(−0.26795, 0)
0
-20


20
(−4, 0)
(2, 0)
-5
0
5
(−1, −9)
(0, −8)
-20


20
(0, 9)
-2
0
(1.5, 0)

7F Applications of parabolas

1.
   a. (4, 7)
   b. (8.83, 0) and (-0.83, 0)
   c. 8.83m
   d.

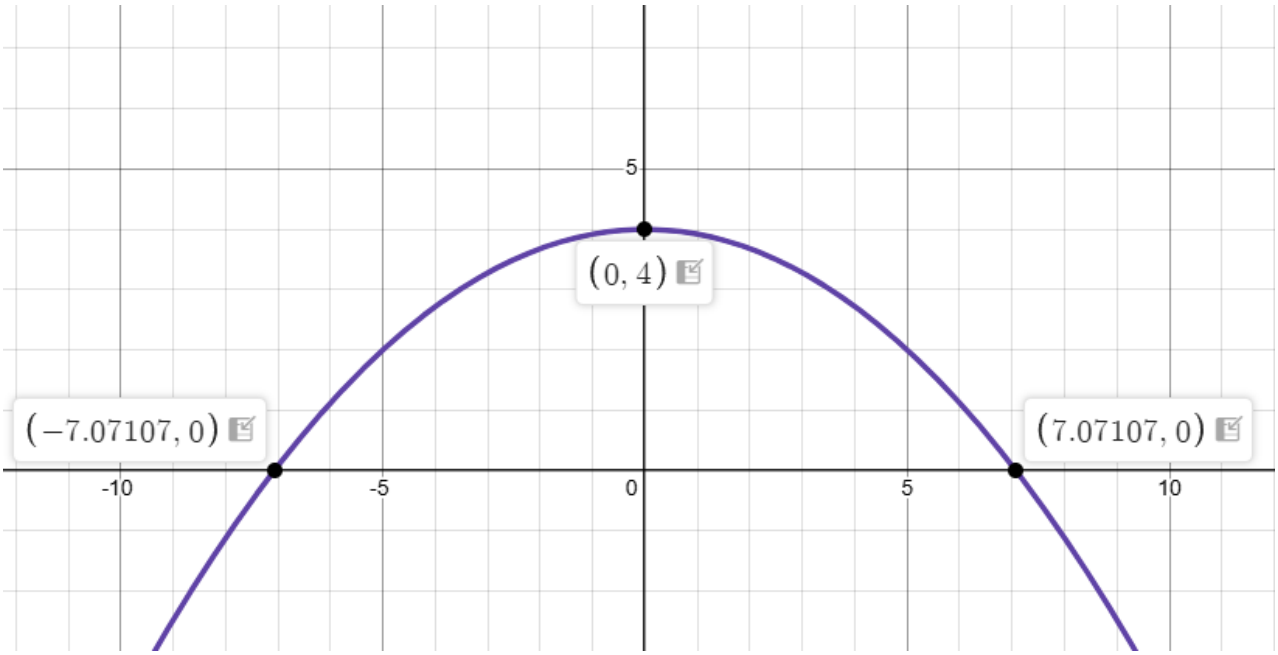


   e. Path spans $0 \le x \le 60$. Ground contact occurs at 8.83 m, height at 6 m is 5.8 m > 0. Yes – it stays dry.

2.
   a. 3.78s
   b. 2m
   c. 18.53m
   d. T=0.517s and 3.156s

3.

a. 14.14m
b. Maximum height: 4m; turning point (0, 4)
c. Y=1.8m, x = (-5.24, 1.8) and (5.24, 1.8);
   $-5.24 \leq x \leq 5.24$
d.



4.

a. $0 \leq x \leq 6$
b. (0, 0) and (6, 0)
c. Maximum height: 4.5m, turning point (3, 4.5)
d. Actual horizontal length = 6 m, **Fails** – exceeds 4 m limit

VSUCCESS 成就教育

7G Intersection of lines and parabolas

1.

$x^2-6x+5=3$	$x^2-6x+2=0 \Rightarrow x=3\pm\sqrt{7}$	$(3+\sqrt{7},\ 3),\ (3-\sqrt{7},\ 3)$
$-2x^2+2x+8=-1$	$-2x^2+2x+9=0 \Rightarrow x=\frac{1\pm\sqrt{19}}{2}$	$\left(\frac{1+\sqrt{19}}{2},\ -1\right),\ \left(\frac{1-\sqrt{19}}{2},\ -1\right)$
$2x^2-4x+1=5$	$x^2-2x-2=0 \Rightarrow x=1\pm\sqrt{3}$	$(1+\sqrt{3},\ 5),\ (1-\sqrt{3},\ 5)$
$-3x^2+12x-7=4$	$3x^2-12x+11=0 \Rightarrow x=2\pm\frac{\sqrt{3}}{3}$	$\left(2+\frac{\sqrt{3}}{3},\ 4\right),\ \left(2-\frac{\sqrt{3}}{3},\ 4\right)$

2.

$x=4-\sqrt{10}\ ,\quad y=10-3\sqrt{10}$

$x=4+\sqrt{10}\ ,\quad y=10+3\sqrt{10}$

$x=\frac{9}{4}-\frac{\sqrt{65}}{4}\ ,\quad y=\frac{11}{4}+\frac{\sqrt{65}}{4}$

$x=\frac{9}{4}+\frac{\sqrt{65}}{4}\ ,\quad y=\frac{11}{4}-\frac{\sqrt{65}}{4}$

$x=1-\sqrt{11}\ ,\quad y=6-2\sqrt{11}$

$x=1+\sqrt{11}\ ,\quad y=2\left(3+\sqrt{11}\right)$

$x=-5\ ,\quad y=-4$

$x=-1\ ,\quad y=4$

3.

2 solutions	2 solutions
0 solutions	1 solution
0 solutions	2 solutions

4.

$x=2-\sqrt{11}\ ,\quad y=11-4\sqrt{11}$

$x=2+\sqrt{11}\ ,\quad y=11+4\sqrt{11}$

$x=\frac{2}{3}\ ,\quad y=-\frac{1}{3}$

$x=1\ ,\quad y=-1$

7H Functions

1.

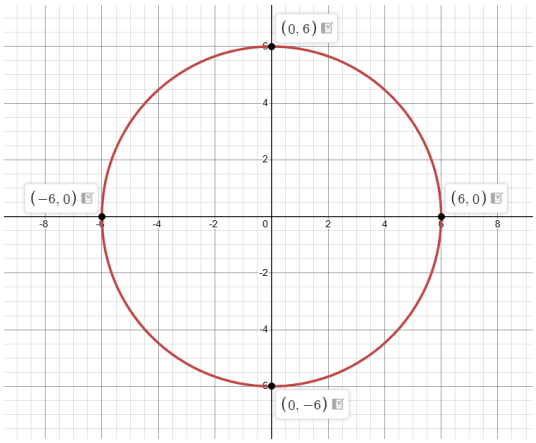
Relation	Reason	Function
$\{(2,5),(3,7),(4,9)\}$	Each x appears once	Yes
$\{(0,1),(1,2),(0,3)\}$	$x=0$ maps to 1 and 3	No
$\{(-2,4),(1,-1),(3,0),(2,4)\}$	All x-values distinct	Yes
$y=5-3x$	Straight line, passes vertical-line test	Yes
$x=4$	Vertical line, fails vertical-line test	No
$y=x^2$	Parabola opening up, vertical-line test OK	Yes
$\{(1,2),(2,2),(3,2)\}$	Distinct xs, same y allowed	Yes
$\{(-1,0),(0,0),(1,0),(1,1)\}$	$x=1$ has two outputs	No
$y^2=x+1$	Sideways parabola ⇒ some x give two ys	No

2.

Function	Limit statement
$f(x)=3x^2-7$	$f(x)\to+\infty$ as $x\to+\infty$
$f(x)=\dfrac{4}{x^2}$	$f(x)\to 0^+$ as $x\to-\infty$ (and as $x\to+\infty$)
$f(x)=5^x$	$f(x)\to 0^+$ as $x\to-\infty$
$f(x)=-2^x$	$f(x)\to-\infty$ as $x\to+\infty$
$f(x)=\sqrt{x}$	$f(x)\to+\infty$ (slowly) as $x\to+\infty$
$f(x)=x^3-4x$	$f(x)\to-\infty$ as $x\to-\infty$
$f(x)=\dfrac{7}{x-5}$	$f(x)\to 0^+$ as $x\to+\infty$
$f(x)=\log(x)$	$f(x)\to+\infty$ as $x\to+\infty$
$f(x)=\log(x)$	$f(x)\to-\infty$ as $x\to 0^+$ (approach from the right)

7I Graphs of circles

1.
   a. (0, 0)
   b. r = 6
   c. $y = \pm 3\sqrt{3}$
   d. $x = \pm 2\sqrt{5}$
   e.

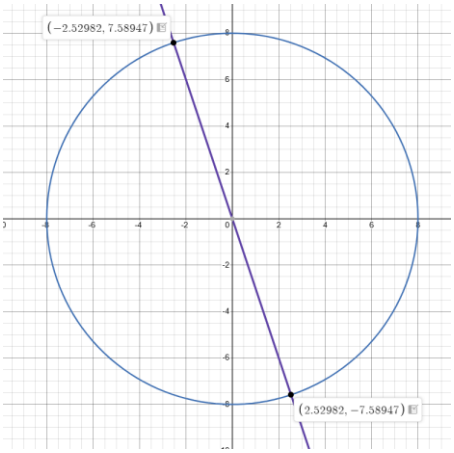


2.
   a.

$$x = -4\sqrt{\frac{2}{5}}, \quad y = 12\sqrt{\frac{2}{5}}$$

$$x = 4\sqrt{\frac{2}{5}}, \quad y = -12\sqrt{\frac{2}{5}}$$

   b.

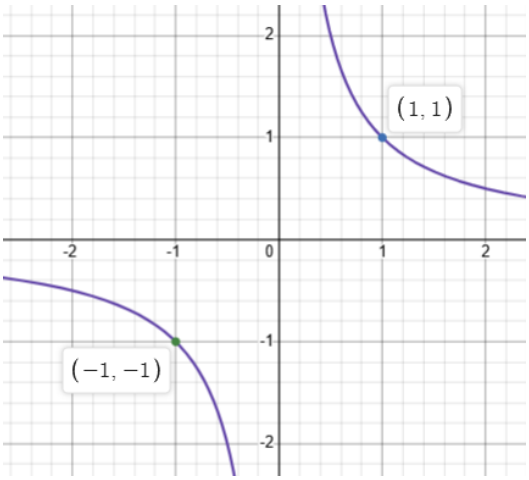


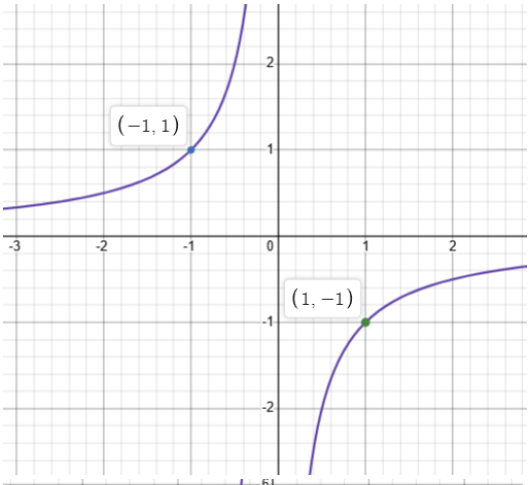
3.

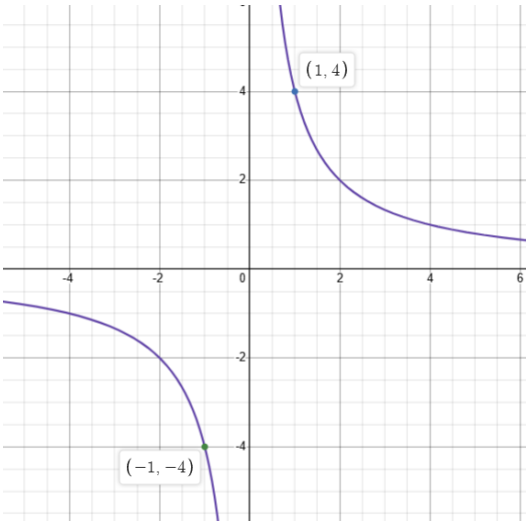
a. $m = \pm\sqrt{3}$
b. $m > \pm\sqrt{3}$
c. $m < \pm\sqrt{3}$

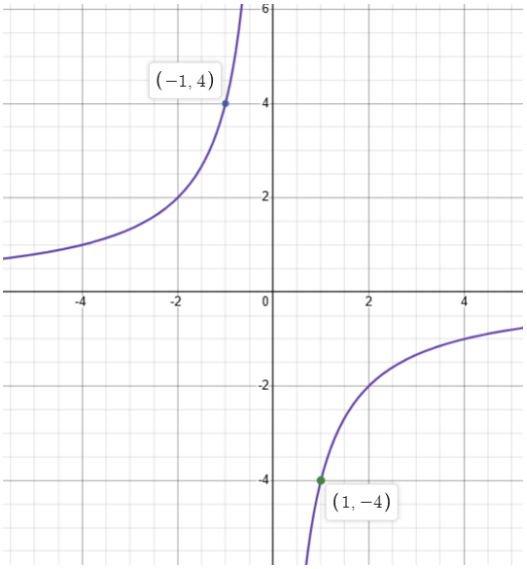
7J Graphs of hyperbolas

1.









2.

$x = -1, \quad y = -1$

$x = 1, \quad y = 1$

$x = -\frac{1}{\sqrt{2}}, \quad y = -\sqrt{2}$

$x = \frac{1}{\sqrt{2}}, \quad y = \sqrt{2}$

$x = -\frac{1}{\sqrt{3}}, \quad y = -\sqrt{3}$

$x = \frac{1}{\sqrt{3}}, \quad y = \sqrt{3}$

$x = -\sqrt{2}, \quad y = -\frac{1}{\sqrt{2}}$

$x = \sqrt{2}, \quad y = \frac{1}{\sqrt{2}}$

3.

$x = -\sqrt{5}, \quad y = -\sqrt{5}$

$x = \sqrt{5}, \quad y = \sqrt{5}$

$x = -\sqrt{\frac{5}{2}}, \quad y = -\sqrt{10}$

$x = \sqrt{\frac{5}{2}}, \quad y = \sqrt{10}$

$x = -\sqrt{\frac{5}{3}}, \quad y = -\sqrt{15}$

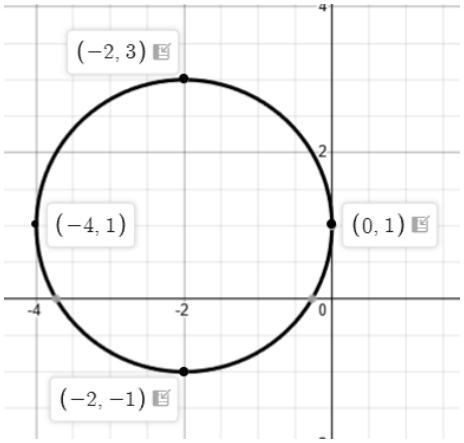
$x = \sqrt{\frac{5}{3}}, \quad y = \sqrt{15}$

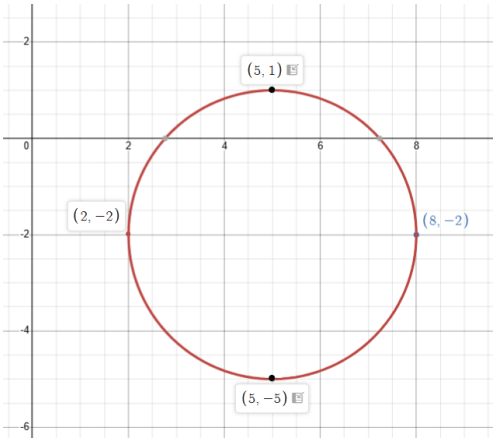
$x = -5\sqrt{2}, \quad y = -\frac{1}{\sqrt{2}}$

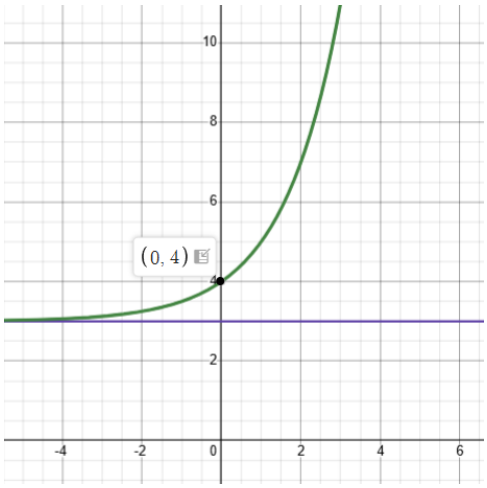
$x = 5\sqrt{2}, \quad y = \frac{1}{\sqrt{2}}$

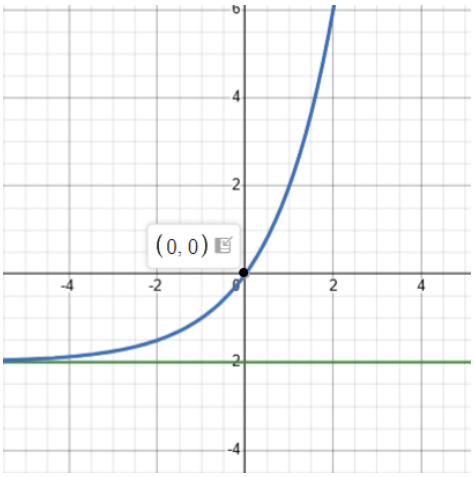
7K Further transformations of graphs

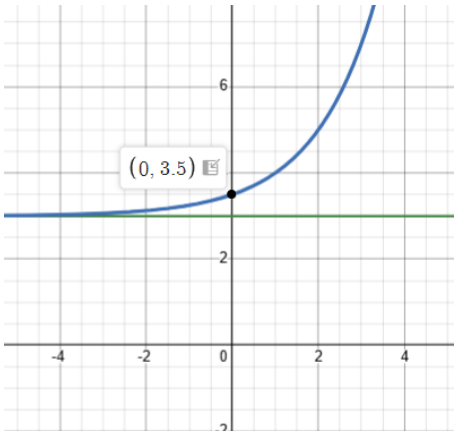
1.

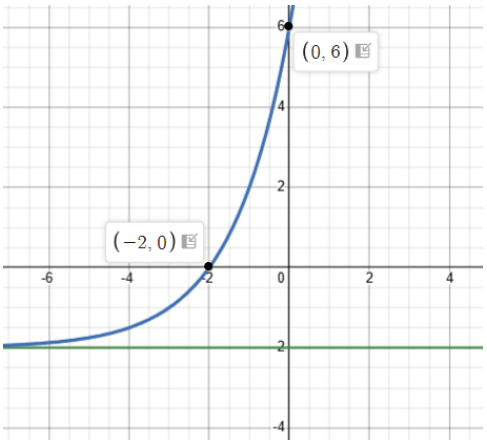


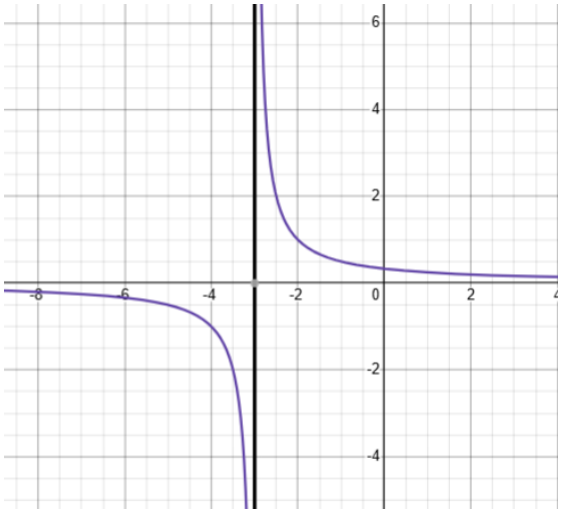


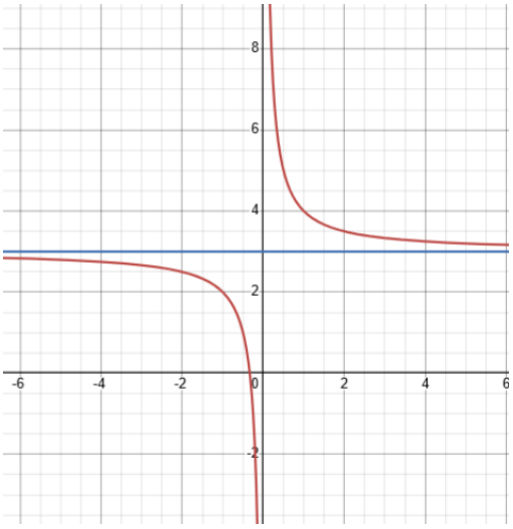









6
4
2
-8
-6
-4
-2
0
2
-2
-4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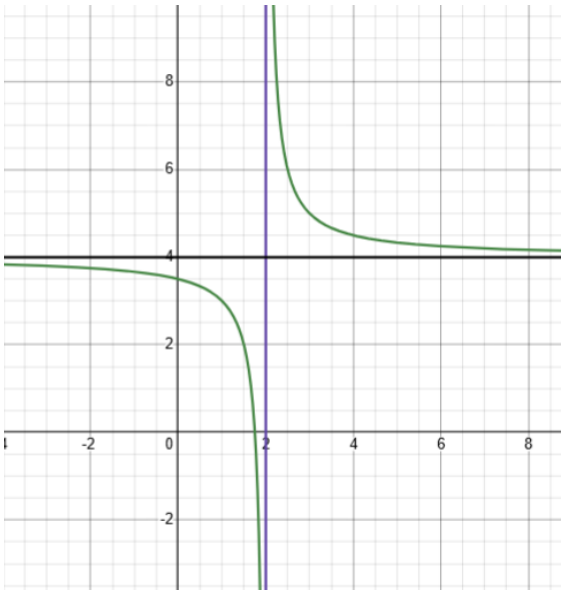
2
-6
-4
-2
0
2
4
6
-2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2
4
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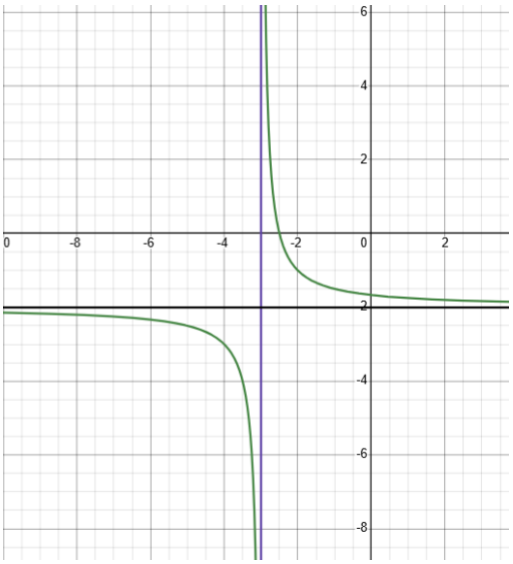
8
-2


6
4
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-4
-2
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2
-2
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